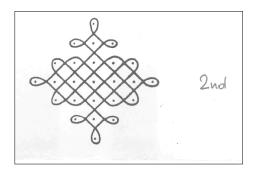
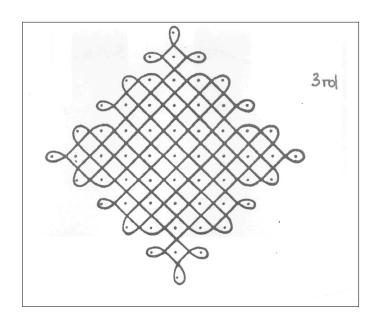
## Mathematics for Elementary Teachers







Recommendations of
The Math for Elementary Education Core Curriculum Subcommittee
of
The British Columbia Committee on the Undergraduate Program in
Mathematics & Statistics

## **Table of Contents**

Committee mandate and members	page 5
I. Guiding Principles	page 6
II. Suggested Course Content	page 7
III. Appendices	
Appendix 1: Interesting Links	page 9
Appendix 2: Textbooks	page 9
Appendix 3: Activities	page 14

# **BCCUPMS Subcommittee on Mathematics For Elementary Teachers Courses**

# Report approved at the 88<sup>th</sup> BCCUPMS meeting May 18, 2010

BCCUPMS Subcommittee on Mathematics for Elementary Teachers Courses was created at the 2007 BCCUPMS meeting at the Yukon College in Whitehorse.

#### **MANDATE**

The mandate of the committee was:

- 1. To explore the issues surrounding the development of effective Math for Elementary Teachers courses, by consulting various groups, including teachers and math education specialists.
- 2. To produce a statement of objectives for Math for Elementary Teachers course(s), and make recommendations for course prerequisites and texts.
- 3. To make recommendations for next stages.

#### **COMMITTEE MEMBERS:**

Steering Committee:

Malgorzata Dubiel (Simon Fraser University), Chair

Wendy Lynne (Capilano University)

Wayne Matthews (Camosun College)

Susan Milner (University of the Fraser Valley)

Susan Oesterle (Douglas College)

#### **Advisory Committee:**

Veda Abu-Bakari (Langara & TRU Open Learning)

Melania Alvarez-Adem (Pacific Institute for the Mathematical Sciences)

Brent Davis (Faculty of Education, UBC)

Bernice Kastner (Simon Fraser University)

Margaret Wyeth (University of Victoria)

#### WIDER CONSULTATION

April 2007: Workshop at the Changing the Culture conference, Vancouver "Mathematics that elementary teachers need to know" Participants included elementary, middle and high school teachers, graduate students, and post-secondary faculty. The discussion was led and the results recorded by Susan Milner and Malgorzata Dubiel.

May 2007: BCcupms professional development session, Whitehorse "Continuing the discussion...Mathematics that elementary teachers need to know". Participants were post-secondary faculty from around BC. The discussion was led and the results recorded by Susan Milner and Malgorzata Dubiel.

October 2007: Workshop at BCAMT fall conference, Richmond: "Looking Backward to Move Forward in Elementary Teacher Ed". Participants were elementary school teachers. The discussion was led and recorded by Susan Oesterle, Susan Milner and Wayne Matthews.

October 2007 – April 2008: Survey of elementary teachers: available at the BCAMT conference and on-line through BCAMT and BCcupms. The survey was designed by Susan Oesterle with suggestions from the committee; Susan Oesterle compiled the results, which are included in this report.

## I. Guiding Principles

Math for Elementary School Teachers is the most important course taught in any mathematics department. It has the potential to shape the mathematical knowledge and understanding of those who will influence the attitudes and develop the abilities of future generations.

Mathematics for teaching is different from the mathematics that is used by other professions. A much deeper understanding of concepts is required to teach them successfully than simply to use them. All instructors teaching a MFEE course need to be aware of this, and of the need to get this idea across to their students, the future teachers.

The course needs to emphasize conceptual understanding to support procedural knowledge, and broaden students' understanding of what mathematics is. Students need to learn to make explicit their implicit understanding of mathematical concepts. The students in a MFET course and the teacher need to work together to generate insight and understanding.

Throughout all school levels, mathematical concepts need to be presented in a way that supports further development at subsequent levels. This is particularly important in teaching elementary mathematics. Therefore, any MFET course needs to make the students aware of how mathematical ideas and definitions are developed sequentially, and how these ideas have evolved historically.

Everybody can learn to understand and enjoy mathematics at a high level. However, performance at a high level requires more than somebody else explaining the concept well; it requires hard and focused work. Students must share the responsibility for high achievement with the teacher. High achievement and competence together foster confidence in one's abilities.

Given that teachers often teach the way that they are taught, MFET should set a high standard for quality teaching, making use of a variety of different approaches to teaching and learning.

The course cannot be taught properly if there is not enough time for students to reflect on the concepts and ideas. While a two semesters course would be ideal, if one semester course is what we can offer, a minimum of 4 hours a week or at least 52 hours is required to cover the material in the spirit of the guidelines.

Given time constraints, the course cannot cover the contents of the entire elementary school curriculum, nor can it prepare future teachers for all future curriculum changes. It is expected, however, that the course will cover the most important concepts and ideas required to teach elementary mathematics curriculum. The course needs to be taught in such a way that the students realize the need for ongoing exploration of mathematical ideas, and develop a desire to continue their mathematical professional development throughout their careers.

## **II. Suggested Course Content**

### **Mathematical Thinking:**

- mental calculation, estimation, number sense
- identification of errors in calculation & reasoning; explanation of the errors
- problem-solving
- algebraic thinking (use of formulae, understanding equal signs)
- concept of symmetry
- appropriate use of the symbols & language of mathematics
- formulation of conjectures & assessment of their reasonableness

## **Core Math Topics**

#### **Numbers & Operations**

- strategies for understanding basic operations on integers, fractions & decimals
- properties of numbers and operations on numbers
- number theory concepts: primes, prime factorisation, divisibility, GCF, LCM
- fractions/decimals/percent; conversions and equivalents

#### **Geometry & Measurement**

- concepts of unit, measure & dimension
- derivation of/rationale for basic area, perimeter & volume formulae
- strategies for conversion of measurements (especially metric)
- definition & properties of geometric objects
- properties of lines, angles & triangles (intuitive development)

#### Language & Reasoning

- Venn diagrams, sets as collections of objects
- mathematical definitions (appropriate precision & rigour)

## Other topics to choose from (to enhance overall goals)

- arithmetic with different bases
- algorithms (history, variety)
- unsolved problems in number theory
- history of numbers
- precision in measurements/calculations
- other geometries
- topics from probability & statistics
- graphs and interpretation of graphs
- basic concepts of set theory
- the difference between inductive & deductive reasoning

## III. Appendices:

## **Appendix 1: Interesting links**

Wayne Matthews created a website with links to recent US reports on Mathematics for Elementary Teaching courses:

http://wmatthews180.googlepages.com/usreports2008

## **Appendix 2: Textbooks**

The committee has spent a considerable time reviewing existing textbooks as well as reports and recommendations on preparation of elementary school teachers. We have not found an ideal book we could recommend as a textbook for a MFET course. Instead, we have designed a book review questionnaire, which was sent to the members of the committee. The responses are included below. They should not be treated as recommendations for a particular book, but, rather, as suggestions how a book can be used to support teaching a MFET course.

(a) Title: Reconceptualizing Mathematics Author: Sowder, Sowder & Nickerson

Publishing Company: W. H. Freeman & Company

Publishing Date: 2008 Edition: Preliminary Reviewer Name/Institution: Margaret Wyeth, University of Victoria

- 1. How long have you used this textbook? Not yet. We will be adopting the text in September.
- 2. What are its strengths? What is the best feature of the book? *Ask me again once we have some real experience with it!*

I wanted a book that would give students the information and guidance needed to replace the math understanding they (in most cases) missed on their way through school, and take them far enough to challenge and test that understanding. That seems to be the aim for these authors. Whether it works or not remains to be seen.

One thing I like that is different from current texts that I have seen is that each chapter has a section called "Issues for Learning". In these the authors often discuss errors and misconceptions that elementary students have. These errors and misconceptions

are, of course, present in many of our students. Perhaps our students will recognize themselves and be motivated to do something about it.

- 3. What are its weaknesses? What is the worst feature of the book? *I don't think it has quite enough "hard" questions that require a level of understanding rather than procedural learning.*
- 4. Would the book facilitate teaching the course in the spirit of the manifesto? Why or why not? *I think so but see my response to #2*.
- 5. Additional comments: *None*

**(b) Title:** Reconceptualizing Mathematics, Parts 1 and 3 **Author:** Judith Sowder, Larry Sowder, Susan Nickerson **Publishing Company:** W. H. Freeman & Company

**Publishing Date:** 2008 Edition: Preliminary

Reviewer Name/Institution: B. Kastner, SFU

1. How long have you used this textbook? 1 year

2. What are its strengths? What is the best feature of the book?

As suggested by the title, it addresses underlying concepts rather than using a "this is how you do it" approach. A lot of attention is given to multiple interpretations and the importance of context in making the mathematics meaningful.

In addition addressing "word problem" difficulties by showing the many ways to interpret the operations of arithmetic, including language considerations, this text devotes more attention to multiplicative reasoning than I have ever seen elsewhere. This makes it possible to address many concepts involving fractions, decimals, percents, and ratios that potential elementary school teachers should understand but generally do not.

The approach to problem solving using quantitative analysis works well by laying groundwork for looking at relationships among quantities in a problem.

Concept development is approached using "Activities" and "Discussion Questions" that the students are expected to carry out, so it is evident that students have to engage the material actively rather than with a mindset of being told what to do.

3. What are its weaknesses? What is the worst feature of the book?

The Activities and Discussion Questions are often open-ended. This might not be of concern in a regular classroom, but has been a problem in the Distance learning course in which I have been using the text.

I would like to see a complete reorganization of the geometry chapters, and hope that the First Edition, due by the end of 2009, might be an improvement. One reason is that numbers arising from measurements are used throughout, but the chapter on "Measurement Basics" comes near the end. For example, similarity is discussed before formal attention is given to area and volume, making it harder for students to see what happens with scale factors for non-linear measurements of similar figures.

- 4. Would the book facilitate teaching the course in the spirit of the manifesto? Why or why not? *I believe it would, because of the emphasis on concepts and understanding, and the importance of these aspects in being able to solve problems.*
- 5. Additional comments: This is a four-part text that was developed in an institution that combines the teaching of mathematics concepts as well as methodology. As such, there is much we have to skip over, either for lack of time or because it is covered in the course on Methods of Teaching Mathematics. While this does create some problems for the course instructor as well as the students, at the same time we do not get questions from the students about why they have been required to take the course.

(c) Title: Modern Mathematics for Elementary Educators

**Author:** Ruric E. Wheeler & Ed R. Wheeler

**Publishing Company:** Kendall/Hunt Publishing

**Publishing Date:** 2005 Edition: 12<sup>th</sup> (new edition now available)

Reviewer Name/Institution: Susan Oesterle, Douglas College

1. How long have you used this textbook? For over 10 years.

2. What are its strengths? What is the best feature of the book?

The best feature of the book is its excellent exercises. Exercises at the ends of the section are separated into three levels ranging from more routine to those involving higher level thinking. Every section has review exercises that help students to integrate different topics. PCR Excursions are included for many sections: these are exercises built around **P**roblem Solving, **Communicating** mathematics and **R**easoning.

Each section is motivated by an introductory problem (whose solution is given at the end of the section), along with a summary overview of the key ideas in the section and the goals (often as they relate to NCTM standards). Explanations in the text are clear but mathematically correct. There are a large number of "Just for Fun" puzzles and occasional historical tidbits.

3. What are its weaknesses? What is the worst feature of the book?

Our biggest difficulty with the book is that there is no representative of Kendall/Hunt Publishing who serves our institution. Desk copies need to be ordered from the U.S., which can take time. The book comes in a paperback edition, which saves on costs, but the binding is not very robust.

Integers are discussed before fractions, but the exercises in the rational number section have very few questions that involve negatives. This allows instructors to teach fractions first, if they choose, but means that they will need to supplement exercises at some point to include operations on fractions with mixed signs.

4. Would the book facilitate teaching the course in the spirit of the manifesto? Why or why not?

Yes. Ideas are presented logically and connections between ideas are brought out in the discussions. The book is written to an audience of future elementary school teachers, so its philosophy is to foster the development of good teachers: those who have a solid conceptual understanding of the mathematics, have the ability to recognize common errors, are good problem solvers and can communicate effectively. The book will continue to provide a good resource for students after they leave the course. It covers far more than can be covered in a one-semester course.

6. Additional comments: I highly recommend this text.

(d) **Title:** Mathematics for Elementary Teachers: A conceptual approach

**Author:** Bennett Jr., A. and Nelson, L.

Publishing Company: McGraw Hill Higher Education

**Publishing Date:** 2007 **Edition:** 7<sup>th</sup> (8<sup>th</sup> edition now available)

Reviewer Name/Institution: Veda Abu-Bakare, Langara College

- 1. How long have you used this textbook? I personally have used this book for the first time in the Spring 2009 term but the department has used various editions of this book for over 10 years.
- 2. What are its strengths? What is the best feature of the book?

This book has many excellent features including one-page math Activities, Problem Openers at the beginning of each Chapter, Spotlights on Teaching with excerpts from the NCTM Standards, Writing and Discussion exercises, and an excellent companion website with virtual manipulatives. The best feature of the book was its examples and illustrations that showed connections to the world around us.

3. What are its weaknesses? What is the worst feature of the book?

My biggest peeve with the book is that it is written for Americans with American units and references to American places and things such as the US debt clock.

4. Would the book facilitate teaching the course in the spirit of the manifesto? Why or why not?

Yes, this book is in keeping with the aims and objectives in the MFEE course. The exposition is good as are the various examples and exercises that reinforce the learning. It is written in a manner that is accessible to the students – a student who so wishes can learn the material quite easily on the topics we did not cover in the course.

5. Additional comments: *This book is workman-like and amply serves the needs of the students in the MFEE course.* 

## **Appendix 3: Activities**

To teach a MFET course in a way consistent with the Guidelines, lecturing is not enough. Various types of activities can help the students to understand the concepts, and introduce them to mathematical thinking. A selection of such activities, contributed by several of our colleagues who have been teaching such courses for many years, is provided below:

## (a) Problem Solving in Math for Elementary Teachers Courses

A great discovery solves a great problem but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive facilities, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery. (George Polya, in How to Solve It)

Mathematics teachers need to be strong problem solvers and also be equipped to support their students' problem solving efforts. With this in mind, problem solving activities in the Math for Elementary Teachers courses should be integrated throughout the course, encourage a diversity of approaches, and provide opportunities for future teachers to reflect on their own problem solving processes.

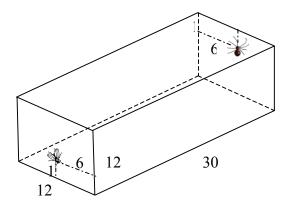
Integration of problem solving throughout the course allows students to see it as a fundamental component of the activity of mathematics. Word problems are often perceived to be difficult leading many students to avoid them, but this is difficult to do if they are incorporated into every topic. Often students' prior negative experiences with mathematics are associated with word problems. Providing frequent problem solving tasks gives students both the opportunity to build confidence in their abilities and to apply and test their conceptual understanding of new ideas in novel contexts.

Although strategies for solving particular types of problems can be learned and applied by rote, the problem solving activities in the Math for Elementary Teachers courses should push students beyond this. The word problems that are given should encourage true critical thinking and analysis, allowing for multiple solution approaches, and should not simply be routine exercises. In a supportive environment, successful experiences with non-routine problems can go a long way towards building confidence. Students should experience both the challenge and joy of problem solving. Furthermore, discussions of alternative solution methods to problems can facilitate insights into mathematics and mathematical reasoning, better equipping students to evaluate and appreciate others' ways of thinking.

Finally, reflection on their own ways of thinking when solving problems should be encouraged for students in the Math for Elementary Teachers context. It provides an opportunity for them to address their own anxieties and allows them to build an awareness of the factors that both facilitate and hinder the problem solving process. This will help them grow both as problem solvers and as future teachers of problem solving. Below are a few favourite Word Problems that can be used in Math for Elementary Teachers courses:

- In a condominium complex for adults only, 3/4 of the women are married to 2/3 of the men. What fraction of the adults in the complex is married?
- Suppose a spider and a fly are on opposite walls of a rectangular room, as shown in the figure below. The spider wants to "visit" the fly, and assuming that the

spider must travel on the surfaces of the room, what is the shortest path to the fly? Be careful! The shortest distance is less than 42!



- You have 16 coins and a balance. Fifteen of the coins are regular and one is a lighter counterfeit coin. Explain how you could locate the counterfeit coin with only three weightings?
- How is it possible to weigh all counting-number masses up to 40 kilograms using only a balance scale and four masses?
- In a college hallway there are 1000 lockers, numbered 1 to 1000. At the beginning all of the lockers are closed. The first student enters and opens all of the lockers. The second student comes along and closes every second locker. The third student goes to every third locker and closes the open ones and opens the closed ones. The fourth student repeats the process with every fourth locker. This goes on until the thousandth student changes the state of the thousandth locker. Which lockers are still open?
- King Arthur has a daughter who is to marry one of his knights. In order to choose he asks all of the knights to sit down around the Round Table. Then he says to the first knight, "You live." To the second knight he says, "You die", and kills him. He continues around the table, allowing the third one to live, condemning the fourth one, and so on, around and around the table until there's only one left. If you know how many knights there are, what position should you sit in to marry the daughter? (Thanks to P. Liljedahl)

## (b) First Day Activities

**(i)** 



## The Tower of Hanoi

-- Susan Milner, UFV

I often use this on the first day of class. Students have told me that it made them think that the course "might not be so bad after all". I made two of these larger models using the rings from Fisher Price toys and enough smaller versions out of cardboard, paper clips and coloured foam so that my classes can work in pairs.

- 1. State the (initial) goal and demonstrate how the rings are allowed to be moved.
- 2. In this class we actually have several goals:
  - find the smallest number of moves when we use 2,3,and 4 rings
  - look for a pattern, predict the smallest number of moves for 5 rings and check it out
  - predict the smallest number of moves for 6 and 7 rings
  - predict the smallest number of moves for 25 rings (or some other large number)
- 3. Students work in pairs. I suggest that they collect their results in a table:

<b>r</b> (rings)	2	3	4	5	6	7	25
<b>n</b> (#							
moves)							

4. After 10-15 minutes, we collect results on the board. Most pairs predict the values for 6 and 7 rings correctly but have a hard time with 25. We compare the patterns the students have discovered. They usually find a pattern based on differences (they increase by 4, 8, 16, etc). Some notice that each value for n is twice the previous one, plus one; I demonstrate how this arises. Very occasionally students will find the pattern that will allow them to predict the number of moves for 25 rings, then n. If so, I have them explain it; if not, we work it out together.  $\binom{2^n-1}{n}$ 

#### 5. Homework:

- Find out who invented this game, when, and what the story was to go with it.
- Answer the original question how long to move 64 rings if each move takes one second?
- Look at some variations: (a) two or three colours, with no two of same colour being allowed to touch, (b) what happens with four spikes? (this leads to a discussion of how easy it can be to ask extremely difficult questions!)
- Make up your own variations.

#### **Comments:**

This activity connects mathematics to games and the very human fondness for patterns. I describe how some version of the game can be played by everyone, from children to mathematicians and computer scientists. In later classes students often report the fun they've had trying it out on their own children.

The class activity demonstrates that different kinds of questions can be associated with the same situation. We also talk about organising data, assigning variables, finding different but equally correct patterns, figuring out how to write down patterns, exponential notation, and generalising results in different ways. We discuss what happens with one ring and how the mystifying  $2^0 = 1$  arises.

## (ii) Two first-day activities

- Jill Britton, Camosun

#### 165

An interesting way to begin establishing mathematics as the study of patterns is by means of simple number tricks. [Allow students use of paper and pencil.]

Think of a number, multiply it by 5, add 6, multiply the result by 4, add 9, multiply the result by 6.

Ask students at random for their result. It will end in 65 if calculations were correct. Subtract 1 from the number preceding the 65 to "guess" the student's number. Accumulate the results (calculation result and number thought of) n the board. Does anyone see a pattern?

Ask the students if they are 100% certain that their rule will work with ANY number. Has the result been "proven" to their satisfaction? If you cross the Pat Bay Highway wearing a blindfold 25 times and don't get hit by a car, have you proven that you cannot get hit by a car if you cross the Pat Bay Highway wearing a blindfold?

To prove the result requires algebra. Slowly reveal steps:

5
5n
5n + 6
20n + 24
20n + 33
100n + 165

#### Card trick

- Have a student select a numerical card (ace, 1-9) at random from a deck of playing cards. Instruct him/her to show the card to the class <u>but not to you</u>.
- Select a second numerical card from the deck, <u>without</u> showing it to the class, and memorize its value.
- Place your card face down on the table. Instruct the student to place his/her card face down on top of yours.
- Tell the class to double their number, add 2 to the result, multiply that result by 5, and then subtract N from the last result, where N = 10 your own number.
- Turn the two cards face up in unison, then fan them so the top card is on the right. The result will be the two-digit number that appears.

To prove the result requires algebra:

a: participantb: magician

a
2a
2a + 2
10a + 10
10a + 10 - (10 - b)
10a + 10 - 10 + b
1-a + b

## (c) Other Activities

## (i) Written Assignment and Group Activity: Magazine Article Summary and Group Project

- Jane Cannon, UFV

#### **Directions for students:**

Choose an article in either *Teaching Children Mathematics* or *Mathematics Teaching in the Middle School*.

Select an aspect of the article that you wish to share with your group. Plan an activity that will take at least 4 but not more than 5 minutes. You may wish to create a worksheet, bring manipulatives so that your group can investigate the concept, devise a model for a demonstration.

Hand in a one page typed report that includes

Magazine title, date, title of the article, author

Brief summary of the article

List of at least two objectives of your group activity

#### **Comments:**

Toward the end of the course students are ready to apply some of the concepts that they have learned. This activity gives them an opportunity to write about mathematics and to engage fellow students in a meaningful activity. Students tend to be quite enthusiastic about their "show and tell" event and gain experience in engaging others in working with mathematical concepts.

## (ii) A variation: Summary & Modifications

- Susan Milner, UFV

#### **Directions for students:**

#### The summary:

Write an accurate summary of the most important points in the article. This should be  $1 - 1\frac{1}{2}$  pages long, double-spaced. Imagine that you are creating a teaching resource for your busy colleagues: it needs to be both accurate and concise. Be sure to identify the article and its source, and be sure to properly credit the author(s) of the article! Attach a photocopy of the article to your work, so I can compare it to your summary.

#### Your modifications:

Think about how you could modify one or more of the ideas discussed in the article; for example, you might modify it for younger or older children, or to teach a related mathematical concept. Use what you know about children and about mathematics. I am looking for implementations that are feasible as well as creative. This should be about a page long.

#### **Comments:**

Students often tell me how surprised to find such interesting articles about teaching mathematics! Also, reading about real teachers using some of the concepts we study reassures them that they are learning something useful in our course. Most find it exciting to think up ways to use a technique or idea for a different audience. I find that this assignment encourages a shift in students' perception of themselves as future teachers.

## (iii) Take Home Test: Partners in Problem Solving

-- Jane Cannon, UFV

#### **Directions for students:**

- Choose two of the following five problems. For each problem select a "partner in problem solving" (not a current Math 105 student).
- Your solutions will be graded on organization, correctness, strategies utilized, reader friendliness and creativity.
- Hopefully your partner will help you explore each problem and devise creative strategies that work toward a convincing and meaningful solution.
- Math communication in a large class setting is not always possible. During this project you will have ample opportunity to discuss, interpret, explain and explore cooperatively.

#### **Comments:**

Often students are quite apprehensive before taking a formal math test. A takehome portion has proved to be helpful in removing some of the math anxiety.

The five problems should be of various levels of difficulty.

#### Sample problems:

Sylvia's Sweet Shop reduced its employees by 2/5 one year and by 1/4 the next year. If 99 employees were left, how many were there originally?

Tim has a collection of nickels, dimes and quarters worth \$2.00. If the nickels were dimes and the dimes were nickels, the value of the coins would be \$1.70. How many nickels could Tim have?

## (iv) Probability Group Assignment Problem

-- Margaret Wyeth, UVic

A suggested instructional strategy for Grades 2-3 in the 1995 Integrated Resource Package for B.C. (p. 50) is to play Counters Away.

'Students have 11 counters that they can place on a number line of 2 to 12. For example, they may place three counters on number 5, four counters on number 8, and one each on 9, 10, 11, and 12. Two dice are rolled. The counters on the number of the sum of the dice are removed. The object is to be the first to have all of his/her counters removed.'

The instructions are a little unclear, but you can assume that there is one dice roll at a time for the whole class. Thus, if the dice show a 3 and a 2 then every student with counters on 5 can remove them. If the next roll is a 4 and a 1 then nothing happens because all the counters on 5 have already been removed. The student who removes all his/her counters first is the winner.

As teacher you are supposed to observe 'to what extent students appear to be calculating possible outcomes based on their understanding of the probability of dice rolls, and to what degree they can describe their strategy for winning the game.'

- (a) Make a list or a table to show the possible outcomes of the dice tosses and give their theoretical probabilities.
- (b) Discuss the effectiveness of each of the following strategies for winning the game:
  - a. Put one counter on each number because then you are more likely to get to remove one.
  - b. Put your counters on the numbers that didn't come up in the last game because the numbers that didn't come up last time are more likely to come up this time.
  - c. Put them all on 7 because 7 has the highest probability of all the sums.
  - d. Put some on 6, some on 7 and some on 8, because 6, 7 and 8 are the sums with the highest probabilities.
  - e. Put them on randomly because the dice will fall randomly.

If you are not very confident with theoretical probability, it is strongly recommended that you actually play the game to help identify the winning strategy - the empirical or experimental method.

## (v) Design your own measurement system

-- Margaret Wyeth, UVic

This should encompass **linear**, **area and volume** measurements and not be an existing system with only the names changed. It should be practical in the sense that it would be possible to use it.

- (a) Present tables showing how your different units of measurement are related. Provide a table each for linear and area measures. You must show a similar one for volume, and if you wish you may include another for a simplified system for volume (like litres within the metric system).
- (b) Give the conversion factors for converting from metric measure to your system, and from your system to metric. You must give a minimum of three: for the metric side use m, m<sup>2</sup>, and m<sup>3</sup>.
- (c) In your system, what would the most suitable units of measure be for the following objects? If practical, make a measurement (marked by an asterisk). If your system has no suitable units for any of these, perhaps your system needs some more work.
  - (1) the height of a tree
  - (2) the side length of the blue square in the header line of your text pages. \*
  - (3) the perimeter of a page in your text \*
  - (4) the distance from the university to Victoria airport
  - (5) the area of grass in the centre of campus
  - (6) the area of a page in your text \*
  - (7) the area of this dot  $\bullet *$
  - (8) the volume of a can of pop \*
  - (9) the volume of cement in a highway overpass
  - (10) the volume of a stack of (i) 10 texts, (ii) 100 texts \*