Comparison: MoE Proposed Curriculum and the Mathematics Proficiencies Report

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MoE Proposed Mathematics Curriculum

. . . the redesigned Grades 10–12 curriculum will support both disciplinary and interdisciplinary learning, encourage locally developed curriculum, and enable a variety of learning environments and school and classroom configurations. This flexibility supports teachers and students wanting to organize learning through interdisciplinary inquiries that focus on project-based learning, problem-based learning, or learning through design.

- The proposal was developed by a high school teachers working group supported by the Ministry of Education.
- Timeline: the new curriculum will have an optional implementation year starting this September; it becomes mandatory in September 2017.
- Click here for a draft of the proposed mathematics curriculum by the Ministry of Education.

Grades 10 and 11: Required Curriculum

All students will be required to take one Mathematics course in each of Grades 10 and 11.

- Mathematics 1 (Grade 10): Foundations and Pre-Calculus, or Workplace.
- Mathematics 2 (Grade 11): Foundations, Pre-Calculus, or Workplace.
  - Pre-Calculus is for the student who intends to continue in a post secondary program which requires Calculus;
  - Foundations is for the student who intends to continue in a post secondary program which does not require Calculus; and
  - Workplace is for the student who intends to enter directly into the workplace.

Optional Curriculum

The courses marked as 10–12 are not restricted to a particular grade level. For example, Geometry could have students from grades 10–12.

- Pre-Calculus 12
- Foundations 12
- Statistics 10–12
- Geometry 10–12
- History of Mathematics 10–12
- Calculus 12
- Apprenticeship 12

Overview of the Curriculum

- Core Competencies: skills which everyone should have; common for all grades, Kindergarten to Graduation.
- Big Ideas: what students should understand or appreciate about the subject. They are unique to each course.
- Curricular Competencies: what students should be able to do, identical except History of Mathematics 12. They are discussed under the following headings:
  - Reasoning and Analyzing;
  - Understanding and Solving;
  - Communicating and Representing; and
  - Connecting and Reflecting.
- Content: what students should know; unique to each course.

Notes.

- The *Curricular Competencies* and *Content* are the *Learning Standards* which teachers are required to teach and assess by the School Act.
- The Curriculum for Mathematics K–9 is in place. These web pages probably show how the curriculm for Mathematics 10–12 will look.

Core Competencies, Kindergarten to Graduation.

- Communication
- Thinking
  - Creative Thinking
  - Critical Thinking
- Personal and Social

Big Ideas: Foundations and Pre-Calculus 10

- Proportional comparisons can be made among right triangles using trigonometry.
- The meaning of each operation including powers, extends to algebraic expressions.
- Rate of change is an essential attribute of linear relations, and has meaning in the different representations, including equations.
- Operations between polynomial expressions are connected and allow us to make meaning through abstract thinking.
- Analyzing simulations and data allow us to notice trends and relationships.

Skip Big Ideas to Curricular Competencies.

Big Ideas: Pre-Calculus 11

- Proportional comparisons can be made among triangles, and angles on a coordinate plane using trigonometry.
- The meaning of each operation, including rational exponents and radicals, extends to algebraic expressions.
- Functions allow us to model contextualized situations including financial ones.
- Operations between algebraic expressions equations are connected and allow us to make meaning through abstract thinking.

Big Ideas: Foundations 11

- Proportional comparisons can be made among triangles and angles.
- Quadratic functions and systems of equations can be represented in many connected ways.
- Logical reasoning helps us discover and describe mathematical truths and counterexamples.
- A statistical analysis allows us to notice trends and relationships.

Big Ideas: Pre-Calculus 12

- Many functions are related through inverse operations.
- Analyzing the characteristics of functions allows one to solve equations, and model and understand relationships.
- Transformations of shapes extends to functions in all of their representations.
- Geometrical thinking and visualization can be used to explore conics and functions.

Big Ideas: Foundations 12

- Combinatorics provides efficient strategies for counting.
- Understanding the characteristics of a variety of functions helps in modelling data.
- Logical reasoning helps us pose, organize and defend arguments.
- Using spatial relationships, we can create, measure and describe objects in geometry.
- Stories can be told using mathematical evidence and reasoning: Counting strategies and organizational tools allow us to calculate probabilities.

Big Ideas: Geometry 10–12

- Working with diagrams is essential to geometric thinking.
- Geometry is about working with variation and invariance.
- Working with and on definitions is central in geometry.
- Geometry stories and applications vary across cultures and time.
- A written proof is the endpoint to the process of proving.

Big Ideas: History of Mathematics 12

- Mathematics has developed over many centuries and continues to evolve.
- The development of number and number systems occurred in many areas of the world and has evolved over time.
- The algebra of today is an accumulation of work throughout the ages by mathematicians from many cultures.
- Tools and technology have progressed mathematics forward.
- Historical mathematicians nurtured a sense of play and curiosity which lead to the development of many areas in mathematics.

Big Ideas: Statistics 12

- Data should be gathered and organized with care in order to answer questions.
- Data can be analyzed using a variety of methods.
- Statistical knowledge is used when working with data to find reliable results that are trustworthy.
- Conclusions can be represented graphically and numerically to communicate and inform.

Big Ideas: Calculus 12

- The concept of a limit is foundational in developing calculus.
- Differential calculus develops the concept of instantaneous rate of change of one quantity in relation to another.
- Integral calculus develops the concept of finding the sum of an infinite series.
- Derivatives and integrals have an inverse relationship.

Curricular Competencies: Reasoning and Analyzing

- (Geometry only) Engage in **spatial reasoning** in a dynamic environment;
  being able to think about shapes (real or imagined) and to mentally transform these shapes to notice relationships.
- Use **reasoning and logic** to analyze and apply mathematical ideas;
  inductive and deductive reasoning, predicting, generalizing, drawing conclusions through experiences including puzzles, games, coding.
- **Estimate** reasonably;
  being able to defend the reasonableness of an estimate; across mathematical contexts.
- Demonstrate **fluent and flexible thinking** of number;
  using known facts, benchmarks, partitioning, applying whole number strategies to rational numbers and algebraic expressions.
- Use **tools or technology** to analyze relationships and test conjectures;
  physical and digital tools including coordinate grids.
- **Model** mathematics in contextualized experiences;
  use concrete materials, dynamic interactive technology, representing a situation graphically and/or symbolically. Link to Three-Act Tasks

Curricular Competencies: Understanding and Solving

- Develop, demonstrate, and apply **conceptual understanding** of mathematical ideas
- **Visualize** to explore and illustrate mathematical concepts and relationships
- Apply **flexible strategies** to solve problems in both abstract and contextualized situations
- Engage in problem-solving **experiences** that are connected to place, story, and cultural practices and perspectives relevant to local First Peoples communities, as well as other cultures.

Skip Curricular Competencies to Content.

Curricular Competencies: Communicating, Representing

- Communicate mathematical thinking in **many ways**
- Use mathematical vocabulary and language to contribute to mathematical **discussions**
- **Represent** mathematical ideas in a variety of ways
- Explain and justify mathematical ideas

Curricular Competencies: Connecting and Reflecting

- **Reflect** upon mathematical thinking
- Use mathematics to support personal choices
- Connect mathematical concepts to each other and to **other areas and personal interests**
- **Incorporate** First Peoples worldviews and perspectives to **make connections** to mathematical concepts.
  Link to Bishop's six "universal" mathematical activities.
  Link to First Nations Education Steering Committee

Curricular Competencies: History of Mathematics 12

Reasoning and Analyzing

- Explore, make connections, predict, analyze, generalize, and make conclusions
- Use historically appropriate tools and technology to explore problems from the past

Understanding and Solving

- Explore multiple strategies used to solve problems throughout history
- Develop, construct, and apply mathematical understanding through play, inquiry and problem solving
- Engage in problem-solving experiences that are connected to place, story and cultural practices relevant to the historical context

Curricular Competencies: History of Mathematics 12

Communicating and Representing

- Communicate in a variety of ways including written and oral language from a variety of cultures
- Understanding historical mathematical understanding through concrete, pictorial, and symbolic representations of the past

Connecting and Reflecting

- Accessing the significance of key figures throughout history and their contributions to the advancement of mathematics
- Explore, apply and connect concepts to each other
- Explore the consequences of mathematic on culture socially and politically
- Incorporate First Peoples principles of learning for example storytelling, learning takes patience and time

Content: Foundations and Pre-Calculus 10

- operations on **powers** with integral exponents;
  positive and negative exponents, exponent laws, evaluating,
  numerical and variable bases.
  C: Understand and use Exponential Expressions.
- **relationships** among data, graphs, and situations;
  communicating domain and range in contextualized situations.
  L: Understand and use the Function Concept.
- **linear relations** including slope and equations of lines;
  one or more types of equations of lines, parallel and perpendicular,
  arithmetic sequences.
  M: Understand and use Straight Line and Linear Functions.
- solving **systems** of linear equations;
  solving graphically and algebraically.
  E: Solve Equations and Inequalities.

Content: Foundations and Pre-Calculus 10

- **multiplication** of polynomial expressions;
  distributive property between two polynomials.
  A: Understand and use Polynomial Expressions.
- polynomial **factoring**;
  GCF, intended for simpler cases involving trinomials and difference of squares.
  A: Understand and use Polynomial Expressions.
- primary trigonometric ratios;
  H: Understand and use Triangle Trigonometry.
- **experimental probability**;
  simulations through play and creating games and connect to theoretical probability where possible.
- **financial literacy**: gross and net pay;
  types of income, income tax and other deductions.

Skip Content to the Mathematics Proficiencies Report.

Content: Pre-Calculus 11

- operations on **powers** with rational exponents and radicals.
  exponent laws, evaluating, numerical and variable bases
  C: Understand and use Exponential Expressions.
- **real number** system
  classification, ordering
- **exponential** functions
  simple exponential functions and their graphs in relation to growth and decay
- **financial literacy**: investments and loans
  investments, loans (lease vs buy), credit cards, mortgages, and graphical representations of financial growth
- polynomial **factoring** of the form $ax^2 + bx + c$, $a^2x^2 - b^2y^2$
  a is not restricted to 1
  A: Understand and use Polynomial Expressions.

Content: Pre-Calculus 11

- **rational** expressions and equations
  simplifying and applying operations to expressions; solving equations algebraically
  B: Understand and use Rational Expressions.
- **quadratic functions** and **quadratic equations**
  characteristics of graphs (including domain and range, intercepts, vertex, and symmetry), multiple forms, and function notation
  factoring, quadratic formula, completing the square
  N: Understand and use the Quadratic Function.
  E: Solve Equations and Inequalities.
- **trigonometry** including non-right triangles and angles in standard position
  sine and cosine law including ambiguous case, angles $0° - -360°$ and special angles
  H: Understand and use Triangle Trigonometry.

Content: Foundations 11

- Mathematical reasoning and **logic**
  conjecturing, inductive and deductive thinking, proofs, and game/puzzle analysis
  U: General, Higher Level or Specialised Proficiencies.
- **Angle relationships**
  properties, proofs, parallel lines and triangles
  F: Understand and use the Geometry of Lines and Points.
- graphical **representations** of quadratic functions
  characteristics of including: end behavior, max/min, vertex, symmetry
  N: Understand and use the Quadratic Function.
- graphical **solutions** to systems of equations
  including linear with quadratic and quadratic with quadratic
  E: Solve Equations and Inequalities.
- solving systems of linear inequalities
  E: Solve Equations and Inequalities.

Content: Foundations 11

- **Trigonometry** with oblique angles
  triangle decomposition, and sine/cosine laws
  H: Understand and use Triangle Trigonometry.
- **Applications** of probabilities and statistics in the real world
  reading about and interpreting surveys with standard deviation, confidence intervals, and distributions
  J: Understand and use Data Analysis.
- mathematics is a tool when conducting **research**
  mathematics used in this course can be used to do a research project of your choice
  U: General, Higher Level or Specialised Proficiencies.
- **financial literacy**: investments and loans
  investments, loans (lease vs buy), credit cards, mortgages, and graphical representations of financial growth

Content: Pre-Calculus 12

- logarithmic functions and equations;
  R: Understand and use the Logarithmic Function.
- **exponential** equations;
  graphing including transformations, solving, base e
- sequences and **series**;
  such as geometric, sigma notation, infinite
  K: Understand and use Sequences and Series.
- operations on **logarithms**;
  laws of logarithms, evaluating with different bases
  R: Understand and use the Logarithmic Function.
- **polynomial** functions and equations;
  solving, factoring, graphing, characteristics of graphs, function notation
  A: Understand and use Polynomial Expressions.
  E: Solve Equations and Inequalities.
  L: Understand and use the Function Concept.
  Q: Understand and use Polynomial Functions.

Content: Pre-Calculus 12

- **transformations** of functions, including $y = \sqrt{x}$, $y = |x|$, $y = \frac{1}{x}$; singular vertical and horizontal expansions, compressions, reflections and translations, inverses, and recognizing composed functions eg. $y = \sqrt{\sin x}$
  L: Understand and use the Function Concept.
- **conics**
  transformations and/or locus derivations;
  P: Understand and use Quadratic Relations.
- **rational** functions;
  characteristics of graphs, asymptotes, intercepts and point discontinuities

Content: Pre-Calculus 12

- trigonometric **functions** and equations with real numbers; radian measure, graphing primary trig ratios including transformations, characteristics, solving
  E: Solve Equations and Inequalities.
  S: Understand and use Circular Trigonometric Functions.
- trigonometric identities;
  using Pythagorean, double angle, reciprocal, sum and difference identities to reduce complexity in expressions
  S: Understand and use Circular Trigonometric Functions.

Content: Foundations 12

- Transformations with iterations to create fractals
- graphical **representations** of polynomial, logarithmic, exponential, and sinusoidal functions
  using characteristics of a graph to identify these functions.
  R: Understand and use the Logarithmic Function.
  S: Understand and use Circular Trigonometric Functions.
- **regressions**
  polynomial, exponential, sinusoidal
- regression **analysis**
  apply the appropriate regression based on the characteristics of data set of graph.

Content: Foundations 12

- set theory and conditional statements
- **combinatorics**
  permutations, combinations, pathways and binomial expansion
- **odds, probability** and expected value
  mutually exclusive, non-mutually exclusive, conditional probability, binomial probability
- mathematics is a tool when conducting **research**
  mathematics used in this course can used to do a research project of your choice
  U: General, Higher Level or Specialised Proficiencies.

Content: Geometry 10–12

- Geometric **constructions**;
  perpendicular, angles, bisectors, triangles, triangle centres, and quadrilaterals
- **Circle geometry**;
  properties of segments, angles, and tangents.
  G: Understand and use the Geometry of Circles.
- **Constructing tangents**;
  lines tangent to circles, circles tangent to circles, circles tangent to three points (PPP), three lines (LLL), etc.

Content: Geometry 10–12

- Transformations of 2D shapes including the **isometries** and **affine transformations.**
  isometries: transformations that maintain congruence (translations, rotations, reflections)
  affine transformations: transformations that maintain collinearity of points and proportions (dilations and shear)
  L: Understand and use the Function Concept.
- **Perspective** and **non-Euclidean geometries**
  perspective: a type of projective geometry where parallel lines meet at a point.
  non-Euclidean: taxicab, hyperbolic/elliptical.

Content: History of Mathematics 12

- Number and Number Systems: including written and oral numbers, zero, rational numbers, pi, irrational numbers, prime numbers
- Patterns and Algebra: early algebraic thinking, variables, early uses of algebra, Cartesian plane, notation, Fibonacci sequence
- Geometry: of lines, angles, triangles, Euclid's Five Postulates, geometric constructions, developments through time
- Probability and Statistics: Pascal's Triangle, games involving probability, early beginnings of statistics and probability
- Tools and Technology: the development over time from clay tables to modern day calculators and computers
- Cryptography: use of cyphers, encryption and decryption throughout history, modern uses of cryptography in war and digital applications

Content: Statistics

- Graphical representations for data
- Statistical analysis of data and relationships between data, including standard deviation and the Normal Distribution, Z-scores, confidence intervals, and correlation co-efficient
- Sampling Techniques and bias
- Formulating hypotheses from data sets
- Statistical techniques to test the validity of hypotheses
- Analyze and make statistical conclusions
- Mathematics is a tool when conducting research

Content: Calculus 12

- Concept of the Derivative
- Limits
- Derivatives
- Applications of Derivatives
- Concept of the Integral
- Integrals
- Applications of integration

A. Understand and use Polynomial Expressions (3.37)

A1	Simplify polynomial expressions	3.96
A2	Add and subtract polynomial expressions	3.96
A3	Multiply and simplify polynomial expressions	3.96
A4	Factor binomials and trinomials	3.83
A5	Factor polynomial expressions of more than 3 terms	3.15
A6	Divide polynomials with binomials of degree 1, using "long division"	3.31
A7	As in A6, using synthetic division	1.40

A. Understand and use Polynomial Expressions (3.37)

Foundations/Pre-Calculus 10:

- **multiplication** of polynomial expressions;
  distributive property between two polynomials. A3.
- polynomial **factoring**;
  GCF, intended for simpler cases involving trinomials and difference of squares. A4.

Pre-Calculus 11

- polynomial **factoring** of the form $ax^2 + bx + c$, $a^2x^2 - b^2y^2$
  a is not restricted to 1. A4.

Pre-Calculus 12

- **polynomial** functions and equations;
  solving, factoring, graphing, characteristics of graphs, function notation. A5.

Skip to Topics which are Missing from the Curriculum

B. Understand and Use Rational Expressions (3.75)

B1	Simplify rational expressions with polynomial numerators and denominators	3.79
B2	Multiply and divide simple rational expressions	3.83
B3	Add and subtract rational expressions	3.83
B4	Simplify complex fractions	3.54

Pre-Calculus 11

- **rational** expressions and equations
  simplifying and applying operations to expressions; solving equations algebraically. B1, B2, B3.

C. Understand and Use Exponential Expressions (3.88)

C1	Use the Laws of Exponents to simplify expressions	3.96
C2	Write radical expressions in exponential form	3.92
C3	Apply the Laws of Exponents to expressions with rational exponents	3.88
C4	Apply Laws of Exponents to expressions with real (including irrational) exponents	3.75

Foundations/Pre-Calculus 10:

- operations on **powers** with integral exponents;
  positive and negative exponents, exponent laws, evaluating, numerical and variable bases. C1.

Pre-Calculus 11

- operations on **powers** with rational exponents and radicals.
  exponent laws, evaluating, numerical and variable bases. C2?, C3.

D. Understand and Use Radical Expressions (3.30)

D1	Simplify radical expressions with index 2. Square roots	3.71
D2	Apply the four basic operations to radical expressions with index 2. Adding, subtracting, multiplying, dividing	3.71
D3	Simplify radical expressions with index n, where n is an integer other than 2	3.21
D4	Apply the four basic operations to radical expressions with index n as in D3	3.17
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2	3.42
D6	Rationalise the numerator or denominator of expressions such as in D5 with index n	2.58

E. Solve Equations and Inequalities (3.23)

E1	Solve linear equations	3.96
E2	Solve linear inequalities	3.85
E3	Solve equations containing absolute value expressions	3.31
E4	Solve quadratic equations by completing the square	3.25
E5	Solve quadratic equations using the quadratic formula	3.81
E6	Solve quadratic inequalities	3.27
E7	Solve equations containing radicals with index 2	3.50
E8	Solve systems of linear equations in two variables	3.71
E9	Graph solutions for systems of linear inequalities in 2 variables	2.67
E10	Solve systems of linear equations in 3 variables	2.56
E11	Solve equations containing rational expressions	3.67
E12	Solve inequalities containing rational expressions	2.79
E13	Solve polynomial equations of degree greater than 2	2.90
E14	Solve polynomial inequalities of degree greater than 2	2.31
E15	Solve logarithmic and exponential equations	3.67
E16	Solve tailored systems of quadratic equations in 2 variables. Few linear terms	2.08
E17	Solve trigonometric equations	3.54

E. Solve Equations and Inequalities (3.23)

Foundations/Pre-Calculus 10

- solving **systems** of linear equations;
  solving graphically and algebraically. E8

Pre-Calculus 11

- **quadratic functions** and **quadratic equations**
  characteristics of graphs (including domain and range, intercepts, vertex, and symmetry), multiple forms, and function notation
  factoring, quadratic formula, completing the square. E4, E5.

Foundations 11

- graphical **solutions** to systems of equations.
  including linear with quadratic and quadratic with quadratic. E16.
- solving systems of linear inequalities. E2, E9.

E. Solve Equations and Inequalities (3.23)

Pre-Calculus 12

- **polynomial** functions and equations;
  solving E13, factoring, graphing, characteristics of graphs, function notation
- trigonometric **functions** and equations with real numbers;
  radian measure, graphing primary trig ratios including transformations, characteristics, solving. E17.

F. Understand, Use the Geometry of Lines, Points (3.18)

F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal	3.25
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles	2.92
F3	Understand and use the Pythagorean Theorem	3.92
F4	Prove simple geometric results	2.65

Foundations 11

- **Angle relationships**
  properties, proofs, parallel lines and triangles. F1, F2, F4.

G. Understand and Use the Geometry of Circles (2.26)

G1	Apply the basic properties of a chord of a circle.	2.13
G2	Apply the basic properties for angles of figures inscribed in a circle. Mainly inscribed triangles.	2.08
G3	Apply some basic properties of tangents to a circle.	2.56

Geometry 10–12

- **Circle geometry**;
  properties of segments, angles, and tangents. G1, G2, G3.

H. Understand and Use Triangle Trigonometry (2.78)

H1	Understand and apply the primary trig ratios as related to triangles.	3.83
H2	Know the trig ratios for special angles as numerical values $(0°, 30°, 45°, 60°, 90°)$.	3.46
H3	Use formulae involving trig ratios for the area of a triangle.	2.40
H4	Use the Laws of Sines and Cosines.	3.10
H5	Understand and apply the concept of a vector.	1.98
H6	Determine the resultant of two vectors.	1.94

H. Understand and Use Triangle Trigonometry (2.78)

Foundations/Pre-Calculus 10

- primary trigonometric ratios; H1

Pre-Calculus 11

- **trigonometry** including non-right triangles and angles in standard position
  sine and cosine law including ambiguous case, angles $0° - 360°$ and special angles. H2, H4.

Foundations 11

- **Trigonometry** with oblique angles
  triangle decomposition, and sine/cosine laws. H4.

J. Understand and Use Data Analysis (0.61)

J1	Organise and graph data from an experiment.	0.88
J2	Obtain, organise and graph sample data.	0.83
J3	Apply sample information to population information.	0.54
J4	Determine 90% confidence intervals.	0.42
J5	Select appropriate sampling methods.	0.38

Foundations 11

- **Applications** of probabilities and statistics in the real world; reading about and interpreting surveys with standard deviation, confidence intervals, and distributions

K. Understand and Use Sequences and Series (2.75)

K1	Understand and use arithmetic sequences and series formulae.	2.75
K2	Understand and use geometric sequences and series formulae.	2.71
K3	Understand and use sigma notation.	2.92
K4	Apply the formula for a geometric series $S = \frac{a}{1-r}$	2.63

Foundations/Pre-Calculus 10:

- **linear relations** including slope and equations of lines;
  one or more types of equations of lines, parallel and perpendicular, arithmetic sequences. K1.

Pre-Calculus 12

- sequences and **series**;
  such as geometric, sigma notation, infinite. K2, K3, K4.

L. Understand and Use the Function Concept (3.70)

L1	Understand and use function notation.	4.00
L2	Identify relations that are not functions.	3.73
L3	Graph basic $\mathbb{R} \to \mathbb{R}$ functions like linear, absolute value, etc. on the Cartesian plane.	3.83
L4	Understand the functional transformations: translation, reflection, expansion with respect to the functions expression.	3.29
L5	Determine the inverse of a function (if it exists).	3.67

L. Understand and Use the Function Concept (3.70)

Foundations/Pre-Calculus 10:

- **relationships** among data, graphs, and situations;
  communicating domain and range in contextualized situations. L3.

Pre-Calculus 11

- **quadratic functions** and **quadratic equations**
  characteristics of graphs (including domain and range, intercepts, vertex, and symmetry), multiple forms, and function notation; L1, L3.
  factoring, quadratic formula, completing the square.

Pre-Calculus 12

- **polynomial** functions and equations;
  solving, factoring, graphing, characteristics of graphs, function notation. L3.
- **transformations** of functions, including $y = \sqrt{x}$, $y = |x|$, $y = {}^{1}/_{x}$; singular vertical and horizontal expansions, compressions, reflections and translations, inverses, and recognizing composed functions eg. $y = \sqrt{\sin x}$. L3.

L. Understand and Use the Function Concept (3.70)

Geometry 12

- Transformations of 2D shapes including the **isometries** and **affine transformations.** L4.
  isometries: transformations that maintain congruence (translations, rotations, reflections)
  affine transformations: transformations that maintain collinearity of points and proportions (dilations and shear)

M. Understand, Use Straight Line/Linear Functions (3.83)

M1	Graph and write the equations for linear functions.	4.00
M2	Understand and use the slope of a line.	4.00
M3	Write direct variations as equations.	3.48

Foundations/Pre-Calculus 10:

- **linear relations** including slope and equations of lines;
  one or more types of equations of lines, parallel and perpendicular, arithmetic sequences. M1, M2.

N. Understand and Use the Quadratic Function (3.36)

N1	Graph and write the equations for given quadratic functions.	3.75
N2	Determine the max. or min. value of a quadratic function. Without calculus.	2.98

Pre-Calculus 11

- **quadratic functions** and **quadratic equations** characteristics of graphs (including domain and range, intercepts, vertex, and symmetry), multiple forms, and function notation; N1. factoring, quadratic formula, completing the square.

Foundations 11

- graphical **representations** of quadratic functions characteristics of including: end behavior, max/min, vertex, symmetry. N2.

P. Understand and Use Quadratic Relations (3.06)

P1	Work with the graph and the equation (+ features) of a circle.	3.31
P2	Work with the graph and the equation (+ features) of parabolas.	3.17
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.	2.85
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.	2.58
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.	3.38

Pre-Calculus 12

- **conics**;
  transformations and/or locus derivations.

Q. Understand and Use Polynomial Functions (2.53)

Q1	Graph and analyse polynomial functions of various degrees (2 and greater).	3.31
Q2	Apply the Factor and Remainder Theorems.	2.48
Q3	Understand and use notation for complex numbers.	1.79
Q4	Analyse the nature of the roots for some polynomial functions.	2.52

Pre-Calculus 12

- **polynomial** functions and equations;
  solving, factoring, graphing, characteristics of graphs, function notation. Q1.

R. Understand and Use the Logarithmic Function (3.59)

R1	Write exponential expressions in logarithmic form.	3.79
R2	Determine values of exponential and logarithmic functions.	3.79
R3	Graph and analyse logarithmic functions of base a where $a > 1$ is an integer.	3.42
R4	Understand and apply the properties of logarithms.	3.67
R5	Use common or natural logarithms to evaluate expressions. Base 10 and base e.	3.63
R6	Perform calculations found in practical settings using the techniques of logarithms. Applications: forgetting, sound, etc.	3.23

R. Understand and Use the Logarithmic Function (3.59)

Pre-Calculus 12

- operations on **logarithms**;
  laws of logarithms, evaluating with different bases. R3, R4.
- logarithmic functions and equations; R4.

Foundations 12

- graphical **representations** of polynomial, logarithmic, exponential, and sinusoidal functions;
  using characteristics of a graph to identify these functions. R3.

S. Understand, Use Trigonometric Functions (3.39)

S1	Understand and use the primary trig ratios as related to angles of rotation.	3.79
S2	Identify reference angles.	3.54
S3	Understand and use the radian measure for angles.	3.79
S4	Understand and use the 3 reciprocal trig functions. $\cot\theta, \sec\theta, \csc\theta$	3.17
S5	Graph and analyse the functions sine, cosine and tangent.	3.79
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.	3.21
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.	2.83
S8	Use the basic trig identities to re-write trig expressions.	3.46
S9	Understand and use sum, difference and double angle identities.	3.38
S10	Prove simple identities.	3.00
S11	Use trigonometric identities to simplify trig expressions.	3.38

S. Understand, Use Trigonometric Functions (3.39)

Pre-Calculus 12

- trigonometric **functions** and equations with real numbers; radian measure, graphing primary trig ratios including transformations, characteristics, solving. S3, S5.
- trigonometric identities; using Pythagorean, double angle, reciprocal, sum and difference identities to reduce complexity in expressions. S8, S9.

Foundations 12

- graphical **representations** of polynomial, logarithmic, exponential, and sinusoidal functions; using characteristics of a graph to identify these functions. S5.

T. Understand, Use Some Concepts of Calculus (1.02)

T1	Use limit notation and determine the limit value of given functions at a point.	1.10
T2	Determine whether a given sequence has a limit.	1.10
T3	Determine the derivative of a given polynomial function.	0.94
T4	Understand and use the power, product and quotient rules.	0.94

Calculus 12 contains more than is suggested here.

U. General, Higher Level or Specialised Proficiencies

U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.	3.54
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.	1.85
U3	Be familiar with writing simple proofs.	2.50
U4	Understand and apply the concept of set, subset and complement of a set.	2.00
U5	Use the set operations: union, intersection.	2.06
U6	Graph set interrelations using a Venn Diagram.	1.52
U7	Use a scientific calculator. Log, Trig, etc.	3.58
U8	Use a graphing calculator.	2.21
U9	Use a computer algebra system. Maple, Mathematica, etc.	1.42
U10	Use a statistical software package. SPSS, Minitab, etc.	0.17

U. General, Higher Level or Specialised Proficiencies

Foundations 11

- Mathematical reasoning and **logic**;
  conjecturing, inductive and deductive thinking, proofs, and game/puzzle analysis. U3.
- mathematics is a tool when conducting **research**;
  mathematics used in this course can be used to do a research project of your choice. U2.

Foundations 12

- mathematics is a tool when conducting **research**;
  mathematics used in this course can used to do a research project of your choice. U2.

Proficiencies of Rank ≥ 3 not in the MoE Curriculum

A1	Simplify polynomial expressions	3.96
A2	Add and subtract polynomial expressions	3.96
A6	Divide polynomials with binomials of degree 1, using "long division"	3.31
B4	Simplify complex fractions	3.54
C4	Apply Laws of Exponents to expressions with real (including irrational) exponents	3.75
D1	Simplify radical expressions with index 2. Square roots	3.71
D2	Apply the four basic operations to radical expressions with index 2. Adding, subtracting, multiplying, dividing	3.71
D3	Simplify radical expressions with index n, where n is an integer other than 2	3.21
D4	Apply the four basic operations to radical expressions with index n as in D3	3.17
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2	3.42

Proficiencies of Rank ≥ 3 not in the MoE Curriculum

E1	Solve linear equations	3.96
E3	Solve equations containing absolute value expressions	3.31
E6	Solve quadratic inequalities	3.27
E7	Solve equations containing radicals with index 2	3.50
E11	Solve equations containing rational expressions	3.67
E15	Solve logarithmic and exponential equations	3.67
F3	Understand and use the Pythagorean Theorem	3.92
L1	Understand and use function notation.	4.00
L2	Identify relations that are not functions.	3.73
L5	Determine the inverse of a function (if it exists).	3.67
M3	Write direct variations as equations.	3.48

Proficiencies of Rank ≥ 3 not in the MoE Curriculum

P1	Work with the graph and the equation (+ features) of a circle.	3.31
P2	Work with the graph and the equation (+ features) of parabolas.	3.17
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.	3.38
R1	Write exponential expressions in logarithmic form.	3.79
R2	Determine values of exponential and logarithmic functions.	3.79
R5	Use common or natural logarithms to evaluate expressions. Base 10 and base e.	3.63
R6	Perform calculations found in practical settings using the techniques of logarithms. Applications: forgetting, sound, etc.	3.23

Proficiencies of Rank ≥ 3 not in the MoE Curriculum

S1	Understand and use the primary trig ratios as related to angles of rotation.	3.79
S2	Identify reference angles.	3.54
S4	Understand and use the 3 reciprocal trig functions. $\cot\theta, \sec\theta, \csc\theta$	3.17
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.	3.21
S10	Prove simple identities.	3.00
S11	Use trigonometric identities to simplify trig expressions.	3.38
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.	3.54
U7	Use a scientific calculator. Log, Trig, etc.	3.58

Proficiencies of Rank ≥ 3, only in Foundations 11, 12

E2	Solve linear inequalities	3.85
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal	3.25

MoE Course Content Not in the MPR

Foundations and Pre-Calculus 10

- **experimental probability**;
  simulations through play and creating games and connect to theoretical probability where possible.
- **financial literacy**: gross and net pay;
  types of income, income tax and other deductions.

Pre-Calculus 11

- **real number** system
  classification, ordering
- **exponential** functions
  simple exponential functions and their graphs in relation to growth and decay
- **financial literacy**: investments and loans
  investments, loans (lease vs buy), credit cards, mortgages, and graphical representations of financial growth

MoE Course Content Not in the MPR

Foundations 11

- **financial literacy**: investments and loans
  investments, loans (lease vs buy), credit cards, mortgages, and graphical representations of financial growth

Pre-Calculus 12

- **rational** functions;
  characteristics of graphs, asymptotes, intercepts and point discontinuities.
- **exponential** equations;
  graphing including transformations, solving, base e.

Pre-Calculus 12

- **conics**;
  transformations and/or locus derivations.

MoE Course Content Not in the MPR

Foundations 12

- Transformations with iterations to create fractals
- **regressions**
  polynomial, exponential, sinusoidal
- regression **analysis**
  apply the appropriate regression based on the characteristics of data set of graph.
- set theory and conditional statements
- **combinatorics**
  permutations, combinations, pathways and binomial expansion
- **odds, probability** and expected value
  mutually exclusive, non-mutually exclusive, conditional probability, binomial probability

MoE Course Content Not in the MPR

Geometry 12

- Geometric **constructions**;
  perpendicular, angles, bisectors, triangles, triangle centres, and quadrilaterals
- **Constructing tangents**;
  lines tangent to circles, circles tangent to circles, circles tangent to three points (PPP), three lines (LLL), etc.
- **Perspective** and **non-Euclidean geometries**
  perspective: a type of projective geometry where parallel lines meet at a point.
  non-Euclidean: taxicab, hyperbolic/elliptical.