MATHEMATICS PROFICIENCIES

For Post-Secondary Mathematics/Statistics Courses

Project Report

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EXECUTIVE SUMMARY

THE MATHEMATICS PROFICIENCIES PROJECT

In Spring, 1998, the Centre for Curriculum Transfer and Technology initiated a project to ***conduct a review of the general and specific mathematics proficiencies recommended by British Columbia's post-secondary educational institutions for entry to selected mathematics/statistics courses***. A Steering Committee was formed to gauge support for such a review in mathematics and to guide the Project. All post-secondary mathematics/ statistics departments would be asked to provide information concerning their general and specific mathematics proficiency expectations of entering students. Similar initiatives in other constituencies were to be scanned.

METHODOLOGY

A Survey questionnaire was created to identify mathematical proficiencies required in first-year *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education* (MFEE). The names of contacts at 24 post-secondary institutions in BC were identified for the Survey. As responses to the questionnaire began arriving, the Steering Committee received summaries of these data and provided helpful guidance.

Analysis of the data included comparisons among the courses in the Survey, ranking the proficiencies, examining them in various groupings, sorting the input on general proficiencies and on specific proficiencies, and organising the examples provided by respondents.

Early on, it became clear that the proficiency expectations for *Calculus* far exceeded those for *Introductory Statistics* or for MFEE. Also, the proficiency expectation profiles for the three courses were quite dissimilar. However, respondents made a universal plea for improved student proficiency in general skills such as multi-step problem-solving and logical thinking.

PROFICIENCIES

The review has resulted in a description of both specific and general proficiencies expected of mathematics students. Lists of specific proficiencies required for entry to each of the three courses in the Survey are available for many future applications. Descriptions of general proficiencies and their importance have also been compiled. Survey evidence of the critical nature of these competencies in surviving post-secondary courses is strong.

Properly used, the results of this Project can become powerful motivators for students preparing for post-secondary courses and programs. These findings can also be used to effect meaningful change in curriculum, pedagogy and evaluation.

Post-secondary departments seeking to refine their mathematical placement or assessment tools will find the details of this Project an invaluable resource.

INTERESTING FINDINGS

Some unanticipated findings and conclusions:

- Students are expected to have strong reading and writing skills in mathematics.
- Working with lines on a Cartesian graph is a most important skill.
- Students are expected to use calculators and read computer printouts in statistics.
- The top Proficiency Category is M (The Straight Line and Linear Functions).
- *Understanding and using the slope of a line* was ranked the overall most important mathematical proficiency.

RECOMMENDATIONS

A brief summary of the recommendations of this Report follows (for more detail see **Recommendations, page 45**):

General Recommendations

- Encourage the development of general mathematical proficiencies for the courses surveyed in this Project.
- Devise instructional modes and evaluation strategies to promote the acquisition of general proficiencies.
- Widely publicise the mathematics expectations of students for post-secondary courses.
- Strengthen communication among provincial groups to enhance student evaluation.
- Urge that forums for the discussion and sharing of common issues be found.

Calculus

- Develop a broad range of strong specific skills along with particular general competencies.
- Expect a clearly developed concept of the function and of graphing techniques.

Introductory Statistics

- Develop a positive attitude towards mathematics and an appreciation of its utility.
- Attain a background in data analysis and a facility with calculators/computers.
- Special expectations include: combinatorics, set theory and solving lengthy problems.
- Strengthen communication among statistics constituencies regarding reasonable expectations of entering students.

Mathematics for Elementary Education

- Make information on the expectations for MFEE widely available.
- Consider increasing proficiency expectations for a larger set of skills than is presently accepted.
- Develop general proficiencies such as logical thinking and comfort with mathematics.
- Encourage a dialogue of stakeholders in MFEE with respect to expectations of students.

PART 1

BACKGROUND and OBJECTIVES

Background of the Mathematics Proficiencies Project

An important task of the Centre for Curriculum, Transfer and Technology (C2T2) is finding ways to improve the transition process for students moving from secondary to post-secondary education. Such transition involves reference to course prerequisites. In order to satisfy entry requirements to a post-secondary course or program, a prospective student presents evidence of having attained a specified or requisite level of achievement in acceptable, previous courses or programs. This is often called **satisfying prerequisites**. Although students satisfy prerequisites, they are often unsuccessful in their chosen post-secondary course or program.

Thus, the principal motivation underlying C2T2's **Entry-Level Proficiencies** initiatives was the suspicion that the existing practice of stating prerequisites in terms of courses and grades was not, in itself, providing sufficient guidance to prospective students and their advisors. While recognising that there are many, varied reasons behind unsuccessful experiences in college, institute or university, it is generally accepted that there is often a discrepancy between the proficiency expectations of post-secondary faculty and the actual skills demonstrated by their students. So, it seemed advisable to explore whether these readiness expectations of faculty could be stated with greater clarity and in more detail. C2T2 chose Business Administration, Health and Human Services, English and **Mathematics** as areas of post-secondary education in which to conduct this exploration.

In early 1998, Proficiency Steering Committees were formed to address the following: endorse a **Proficiency Project** in their area, select a contractor for the Project, assist in the direction of the Proficiency Project, act as a sounding board (focus group) for issues relating to the Project and approve the project's final Report (particularly, its recommendations). Following the establishment of the **Mathematics Proficiencies Steering Committee**, C2T2 elicited Mathematics Proficiencies Project proposals from the post-secondary education community. The Request for Proposals indicated that the Project's final report was to include a description of the **general and specific proficiencies** recommended by post-secondary faculty for admission to first-year mathematics/statistics courses. Further, it stated that subsequent outcomes from the results of the Mathematics Proficiencies Project could be the provision of clear, user-friendly information for secondary students, teachers and curriculum designers about mathematics proficiency expectations in the post-secondary educational system.

At its Meeting of May 21, 1998, the Mathematics Proficiencies Steering Committee selected the contractor for the Mathematics Proficiencies Project and directed that the Project should explore post-secondary proficiency expectations relative to *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education*. The Steering Committee felt that, among all of the usual first-year courses in mathematics and statistics, these three courses would provide suitable diversity and sufficient universality for the exploration.

Goal of the Mathematics Proficiencies Project

The overall goal of the Project was *to conduct a review of the general and specific mathematics proficiencies recommended by British Columbia's post-secondary educational institutions for entry to selected mathematics/statistics courses, and to prepare a report of the findings and recommendations resulting from this review.*

Objectives of the Mathematics Proficiencies Project (Summary)

Under the guidance of the Steering Committee, the specific Objectives of the Mathematics Proficiencies from Secondary for Post-Secondary Mathematics/Statistics Courses Project were:

1. To design and prepare a questionnaire to obtain feedback from British Columbia's post-secondary mathematics/statistics community concerning its specific/general mathematics proficiencies expectations of students entering *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education* courses.

2. To manage the distribution of the questionnaire and the collection of the resulting data/information.

3. To record, collate and analyse the questionnaire responses concerning the specific and general mathematics proficiencies expectations that the post-secondary community has of students entering the specified courses.

4. To work as closely as possible with the members of the Mathematics Proficiencies Project Steering Committee, and to follow their direction and suggestions.

5. To prepare a complete report consisting of the results of the questionnaire, the findings from other sources, the conclusions drawn, and the recommendations made by the Steering Committee that flowed from this review.

A major goal of the secondary mathematics curriculum should be to develop symbol sense.

The transition from elementary to secondary mathematics is characterized by a shift from concrete objects to abstract symbols. Developing fluency with symbols and other abstract entities—which can be geometric, algebraic, or algorithmic—must be a central aim of secondary school mathematics.

Reshaping School Mathematics, p. 45.

THE STEERING COMMITTEE and PROJECT MANAGEMENT

Steering Committee

A preliminary Steering Committee to *Look at Entry-Level Proficiencies, Skills and Attitudes for First-Year Mathematics Courses* first met on March 13, 1998. The Committee, chaired by John Meagher, had members from the following: Adult Basic Education, BC Association of Mathematics Teachers, BC Committee on the Undergraduate Programme in Mathematics (BCcupm), Curriculum Branch (Ministry of Education) and Mathematics/Statistics Departments (University/ Colleges). The intent, initially, was to use the university colleges as a focus for the study.

Upon recommendation of the BCcupm, the Steering Committee for the Mathematics Proficiencies Project was expanded to include members from university and college mathematics/statistics departments. The Mathematics Proficiencies Project Steering Committee (Appendix E), among other duties, served as advisory body to the Project, to approve the findings and recommendations of the Project and as advisory body on "next steps" regarding the results of the Project.

Including the initial meeting, the Steering Committee met on five occasions. In order to assist in expediting meeting arrangements and in eliciting feedback on Project-related matters, members made use of an electronic-mail listserv.

Project Management

The Mathematics Proficiencies Project is an initiative of the **Secondary to Post-Secondary Transitions Team** of the Centre for Curriculum, Transfer & Technology (C2T2). Secondary to Post-Secondary Transitions, which is overseen by **John Meagher** (Senior Program Manager), comprises activities in the areas of Articulation, Career Pathways, Applied Academics and Defining Entry-Level Proficiencies. *Defining Entry-Level Proficiencies* involves working with the post-secondary sector in efforts to identify the basic proficiencies and skills students need for entry to specific, post-secondary disciplines or programs. John Meagher has responsibility for the operational management of all aspects of Secondary to Post-Secondary Transitions and for the Mathematics Proficiencies Project.

Besides being available for consultation and advice, John Meagher established a Steering Committee, consulted with the post-secondary Mathematics/Statistics community and oversaw the contract details of the Mathematics Proficiencies Project.

THE PROCESS

The Steering Committee

At its first Meeting, March 13, 1998, the Steering Committee gave support to the initiation of a proficiencies project in mathematics/statistics. The Committee recommended that John Meagher introduce the Mathematics Proficiencies initiative to the BCcupm's Meeting, May 8, 1998. Following the BCcupm's recommendation, the Steering Committee welcomed additional members from mathematics/statistics departments in universities and colleges.

The following are important recommendations made early by the Mathematics Project Steering Committee: that proficiency expectations in *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education* should be examined in the Project, that Leo Neufeld be asked to conduct the Project, that the Project consist of a questionnaire to post-secondary instructors, a consideration of the use of student focus-group(s) and a search of available literature on the subject of proficiencies for post-secondary mathematics/statistics courses.

As Survey questionnaire responses became available, the Steering Committee provided excellent advice, direction and insight. On September 4, 1998, the Committee recommended following-up with some respondents where oversights might have occurred and advised not to weight responses by institution in any way. Because of time constraints and concerns about the logistics of arranging such events, the Committee advised against either student focus-group sessions or interviews.

In subsequent meetings, the Committee focussed on the key issues raised by Survey responses and on the need to identify general proficiencies along with the particular ones. The Committee strongly urged that the Project Report emphasise the enunciation of and need for general, as well as particular, mathematical proficiencies by entering mathematics/statistics students. Through successive meetings, it became clear that the Committee was also serving as an informal, but very valuable, focus-group for the Project.

The Survey Process

The design and development of the Survey questionnaire involved the use of existing secondary school curricula, guidelines from the Project Steering Committee and personal expertise gained through overseeing and delivering college preparatory courses. After having established a list of university/college/institute contacts for the Survey, the completed questionnaire forms (*Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education*) were distributed to each of 24 post-secondary educational institutions in British Columbia. An Information Sheet accompanied the questionnaire.

By the end of August, 1998, about half of the responses had been received and analysing these early results was possible. Reminders were sent to those contacts from whom returns had not been received. Responses continued to come in through the remainder of 1998 with the last being received at the end of January 1999—signalling a major success (100% return rate)!

Throughout the fall of 1998, summary reports of Survey data available were prepared for the Mathematics Proficiencies Project Steering Committee. Though not complete, the data set

provided for the Steering Committee an increasingly clearer picture of post-secondary expectations with respect to mathematics proficiencies.

The Report

As a part of the Report writing process, preliminary Recommendations were refined by the Steering Committee. The final Project Report writing began in December 1998. By this time, most of the Survey responses had been assimilated, the proficiencies-related literature was available and a clear direction from the Steering Committee had emerged.

In British Columbia, secondary school curricular changes in mathematics, at various times, have included: removing geometry, adding calculus, encouraging the use of technology, using a strands approach, adding data analysis topics and adopting an applications emphasis. Each of these minor or major alterations to the kind of mathematics and manner in which it is taught have responded to a societal influence or a pedagogical imperative. In turn, each has come about through considerable effort, often compromise and usually expense. The effects, whether beneficial or not, can be long-lasting, sometimes permanent.

For this reason, the recommendations written in this Report are made thoughtfully with the full knowledge that, once again, modifications are being suggested.

THE SURVEY

A major objective of the Mathematics Proficiencies Project was to "distribute a comprehensive questionnaire to all post-secondary mathematics departments requesting information concerning their general and specific mathematics proficiency expectations of secondary students." With the assistance of the Steering Committee for the Project, the following guidelines for the design and distribution of this Survey questionnaire were established:

- First-year courses in *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education* would be surveyed.
- The various first-year *Calculus* courses would not be distinguished in the survey and *Introductory Statistics* was to have a non-calculus prerequisite.
- As possible, all post-secondary mathematics departments should be asked to participate in the Survey.
- Opportunity for detailed as well as general input should be given to respondents.

With these guidelines in mind, the Survey questionnaires were created containing Part A, consisting of specific proficiencies to be given an importance rating, and a Part B to be used for input on general proficiency expectations and for offering examples of problems illustrating these and other specific expectations. Specific proficiencies were grouped into Proficiency Categories as seen in the example of Category K below:

Category Code	Proficiency Descriptor	Comment	Importance Rating
K	**Understand and use Sequences and Series.**		
K1	Understand and use arithmetic sequences and series formulae.		
K2	Understand and use geometric sequences and series formulae.		
K3	Understand and use sigma notation.		
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	

The three questionnaires, one for each of the courses being surveyed, were identical except for distinguishing labels. In each, respondents were asked to rate the Proficiency Category on a scale of **4** (Essential) to **0** (Not Important) and to do the same for the individual sub-proficiencies within the Category. They were also asked to write in additional proficiencies which they felt were not covered by the given Descriptors.

Survey questionnaires, one for each of *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education* (MFEE), were mailed to 24 post-secondary educational institutions in British Columbia and responses from each of these have been received.

Making subsequent contact with a few respondents was necessary to clarify the intent or the details of some responses and four institutions were subsequently asked to complete the MFEE questionnaire for other courses at these institutions which transfer as MFEE courses.

FINDINGS and CONCLUSIONS

Proficiencies, whether called abilities, competencies or skills, can be categorised in several ways. However, the practice in other constituencies, the convention followed by most education theorists and the direction given early by the Mathematics Proficiencies Steering Committee all pointed to using a two-tier model, **General** and **Specific**, for identifying the expectations of post-secondary institutions. A complete listing of General Proficiencies does not exist, nor is it possible to establish how basic Specific Proficiencies should be, but those Proficiencies arising here are thought of as belonging to one or the other category.

General Proficiencies - Survey Findings

The information gathered from the Survey concerning the expectations of general proficiencies by post-secondary education faculty of entering students to *Calculus, Introductory Statistics* and *Mathematics for Elementary Education* is presented in anecdotal form. In the Survey questionnaire (Part B), respondents were asked to "**indicate the particular mathematical proficiency or proficiencies that would improve**" students' performance in the courses surveyed. Here, respondents offered their suggestions regarding specific proficiencies that would improve student performance in post-secondary mathematics/statistics courses, but they often also included comments about general weaknesses with recommendations for improvement.

There is an underlying feeling among college/university instructors that overall the secondary mathematics curriculum, in the main, contains the content and topics with which a mathematics/ statistics student would need familiarity in order to succeed in most post-secondary environments. However, the success rates, especially in *Calculus*, are evidence that to judge student preparedness solely on the curricular content of a prerequisite course is unwise. In a full analysis of the reasons for student success, other influences and other mathematical proficiencies must be considered. Influences such as financial need and the stability of personal relationships are beyond the mandate of this Project. Nonetheless, the importance of general mathematical proficiencies will be stressed.

In mathematics education literature, lists of General Proficiencies usually contain some of the following:

- *Inference* – logical deduction and extracting conclusions from various forms of data.
- *Generalisation* – formulating general principles from particular cases.
- *Modelling* – representing problem situations or known constructs in a symbolical or visual manner.
- *Optimising* – finding the best or optimum solution, or the best strategy to solve a particular problem or dilemma.
- *Abstracting* – to think of an object or a principle independent of any instance or specific application.
- *Symbolism* – using symbols to represent concepts for purposes of manipulation and communication.

Although the Report uses some of these terms, we have included others, less general, such as *Problem Solving*, which refers to a level of comfort in solving and presenting the solutions to word problems, and *Positive Attitude towards Mathematics*.

Respondents stated (see **Summary** table below) that thinking skills, language skills, model building, ability to combine concepts, abstracting, generalising, problem solving, comfort with

mathematical symbolism and attitude towards mathematics are proficiencies vital to the student's survival in a post-secondary mathematics/statistics course. Each general proficiency listed in the table is derived from the Survey responses. Faculty deem these proficiencies to be very important to first-year students in the courses surveyed.

Summary of GENERAL Proficiencies from the SURVEY

General Proficiency	Related Proficiencies	Survey Source	Excerpts from Survey Comments
Thinking Skills	Critical, Logical Thinking	Calculus Stats MFEE	…do more word problems…so they develop logical thinking! …they must be able to think critically, analytically… One shortcoming…their problem solving/logic skills. Logic often proves a stumbling block.
Language Skills	Reading Writing	Stats	[Reading and Writing English] – vital skills. …learning to express oneself accurately…verbal proficiencies. …English mastery is a better predictor of success…
Positive attitude towards Mathematics	Unafraid of Mathematics Comfort with Numbers No Anxiety Willingness to Try	Stats MFEE	A general obstacle…the myth that "probability/statistics is hard". Some students enter…with a lot of anxiety. …arrive with a fear of numbers…likely to be unsuccessful… …to overcome the fear and hatred of mathematics… …ability and effort are much more likely to predict success…
Model Building	Concept Development General Strategies	Calculus	…difficulties I see arise…from an over-emphasis of recipe-learning rather than understanding strategies… …I think that function modelling (building…) should serve as the central theme in the secondary school curriculum.
Combining concepts		Calculus	…often need to be able to juggle two or more "pieces" of information… …the most basic concepts of calculus…involve managing complex "collections" of information.
Abstracting		Calculus	…students have trouble translating this as an inequality… …difficulty with algebraic abstractions, such as $\Delta f / \Delta x$ …
Generalising	Seeing common characteristics	Calculus	Another practice that seems to be missed…is the mathematicians' propensity for abstracting and generalising.
Problem Solving		Calculus Stats	Taking an applied problem;…students have very little experience…no skill in attacking problems. Students should do more word problems… Tied to this is a fear/dislike of word problems. …a strong background in translating word problems into math equations should be helpful.
Symbolism		Stats	…be familiar in a general sense with mathematical symbolism…

For ***Calculus*** courses, the Survey comments emphasise that thinking skill, model building, combining concepts, abstracting, generalising and problem solving competencies are General Proficiencies without which students will find these courses a struggle. Respondents also report that an observable number of entering *Calculus* students have real weaknesses in these areas. There is a feeling that action should be taken to assist students in raising their level of **general** mathematical proficiency.

The Survey comments regarding ***Introductory Statistics*** courses point to a requirement for strengths in thinking, English language, problem solving and a positive attitude towards mathematics as general proficiencies expected in statistics courses. Many of the problems students address and the solutions they submit entail lengthy descriptions and complex data sets. In order to analyse such problems and to respond adequately to the questions posed, students are expected to be capable users of language. Respondents, both explicitly and implicitly, underscored the importance of this general proficiency in learning statistics.

Mathematics for Elementary Education (MFEE) courses require thinking skills and a positive attitude towards mathematics as overall general proficiencies. MFEE courses are offered in under 70% of BC's post-secondary institutions, making the volume of Survey comments on MFEE proficiency expectations smaller than that of the other two courses in the Survey. However, respondents expressed frustration with the negative attitude (sometimes hatred) towards mathematics with which many MFEE students enter the course. Although not recorded in the table (page 13), respondents also voiced the wish that entering MFEE students possessed a greater level of comfort with some specific mathematical proficiencies. One respondent captures it best, "*A number of proficiencies that I rated with a high score are actually things we teach in the course. If a student is missing one or two, they can learn them as we go, but it they are missing too many, they are doomed*."

General Proficiencies - Steering Committee

During the course of the Project, the Mathematics Proficiencies Project Steering Committee discussed the subject of General Proficiency Expectations in first-year mathematics/statistics courses on numerous occasions. Although linkages between the comments of Committee members and specific courses of the Survey were not always intended, it is important to note the general proficiency emphases and themes that emerged.

Acting in many respects like a focus-group, the Steering Committee members voiced many of the frustrations about proficiency deficits noted by their colleagues in the Survey:

"*Students think they know the material*", "*the notions of multi-step problems and of synthesising knowledge must find their way into the Report*" and "*students with a superficial understanding of the concepts covered in Math 10, 11, 12 struggle with university calculus and many of these students fail calculus*."

The Committee seemed agreed that, allowing the importance of specific proficiencies, the single most important mathematical shortcoming of students entering post-secondary is their lack of **general** skills. The Committee emphasised thinking, synthesising, handling multi-step problems, solving word problems, modelling, integrating of topics, using symbolism and having a positive attitude towards mathematics as vital general proficiencies for prospective, post-secondary mathematics/statistics students.

In the **Summary of General Proficiencies** table below, the areas of emphasis from the Steering Committee are highlighted. Comments at Committee meetings were often expressions of general insight, rather than applicable to a particular course. Thus the Course Intended column reflects a 'best-guess' in this regard.

Summary of GENERAL Proficiencies from the STEERING COMMITTEE			
General Proficiency	**Related Proficiencies**	**Course Intended**	**Excerpts from Meeting Notes**
Thinking Skills	Critical, Logical Thinking	Calculus Stats MFEE	Central issues are whether thinking skills have been developed and whether sound concept development has occurred.
Synthesising Concepts	Concept Development	Calculus Stats MFEE	Concept development is important. It is important that secondary school evaluation include the synthesis of mathematical ideas. Synthesising knowledge must be mentioned in the report.
Multi-step Problems		Calculus	The matter of multi-step problems needs to be addressed. The notion of multi-step problems must find its way into the report. In secondary school, the final examinations should contain fewer questions and more multi-step problems.
Word Problems		Calculus Stats	Students encounter many word problems—proficiency in this area is very important. Both algebra skills and problem solving skills are needed.
Modelling	Re-stating Problems	Calculus Stats MFEE	Need function construction for word problems addressed.
Positive Attitude towards Mathematics	No math anxiety	MFEE Stats	Students we get are not ready for/hate/are afraid of mathematics. Students don't know what they don't know.
Integration of topics		Calculus	Be able to apply inverse functions with logarithms. Students tend to have memorised techniques or recipes for answering math questions.
Symbolism	Symbol Manipulation	Calculus Stats MFEE	Comfort with symbol manipulation is vital in secondary mathematics/statistics courses.

Overall, Steering Committee members felt that students who do not succeed in post-secondary mathematics/statistics courses tend to have a superficial knowledge of the expected proficiencies and fail to appreciate their lack of preparation at the outset. Members also expressed their concern about an over-utilisation of "recipe learning", as opposed to the development of strategies and proficiencies in synthesising knowledge and for tackling multi-faceted, multi-step problems.

Specific Proficiencies - Survey Findings

Calculus

> Students of calculus must know certain facts and relationships, even if they are readily available from a calculator or computer algebra system, just as a writer is expected to know facts and relationships readily available from a dictionary or the computer's word processing system. There are circumstances in which failure to utilize a reference or the tools afforded by modern technology would be a mark of incompetence. There are other circumstances, however, in which undue reliance on such tools slows one down or even prevents participation with co-workers who assume a certain level of fluency in one's field.
>
> A.W. Roberts, Calculus, The Dynamics of Change, p.3.

The responses to the Mathematics Proficiencies Survey questionnaire showed that all 24 post-secondary institutions in BC have at least one *Calculus* course. Although many have more than one such course, the Survey asked respondents (see page 69) to rate the listed Mathematics Proficiencies as they would apply to all of their first-year calculus courses. Hence, the Survey results for *Calculus* are considered as applying to a generic calculus course.

When appraising the importance of the sub-proficiencies within the Proficiency Categories of the Survey, respondents rated 60% of individual proficiencies to be **Essential to Quite Important** for *Calculus* and another 25% to be **Quite Important to Marginally Important** (see Appendix B, page 56). The proficiencies of *understanding and using function notation*, *graphing and writing equations for linear functions*, and *using and understanding the slope of a line* were unanimously rated as **Vitally Important** to entering *Calculus* students.

Of the eighteen Proficiency Categories in the Survey, respondents rated 78% of them as **Essential to Quite Important** for *Calculus*. The Ranking table (see page 17) displays the order of importance of the Proficiency Categories using the overall mean, shown in column 3, of the rating respondents gave to each Proficiency Category. Column 4 shows the mean of the ratings for sub-proficiencies within each Proficiency Category.

Proficiency Categories Ranking – CALCULUS

Category Code	Proficiency Category Description	Overall Importance Rating	Mean of Ratings for Sub-Proficiencies in Category
L	Understand and use the Function Concept.	**4.00**	**3.70**
A	Understand and use Polynomial Expressions.	**3.96**	**3.37**
C	Understand and use Exponential Expressions.	**3.96**	**3.88**
M	Understand and use Straight Line and Linear Functions.	**3.96**	**3.83**
E	Solve Equations and Inequalities.	**3.92**	**3.23**
S	Understand and use Circular Trigonometric Functions.	**3.92**	**3.39**
B	Understand and use Rational Expressions.	**3.83**	**3.75**
H	Understand and use Triangle Trigonometry.	**3.83**	**2.78**
N	Understand and use the Quadratic Function.	**3.83**	**3.36**
R	Understand and use the Logarithmic Function.	**3.79**	**3.59**
D	Understand and use Radical Expressions.	**3.75**	**3.30**
F	Understand and use the Geometry of Lines and Points.	**3.50**	**3.18**
Q	Understand and use Polynomial Functions.	**3.42**	**2.53**
P	Understand and use Quadratic Relations.	**3.29**	**3.06**
K	Understand and use Sequences and Series.	**2.75**	**2.75**
G	Understand and use the Geometry of Circles.	**2.65**	**2.26**
T	Understand and use some Concepts of the Calculus.	**1.06**	**1.02**
J	Understand and use Data Analysis.	**0.79**	**0.61**

Many of the mathematical proficiencies listed in the Survey questionnaire relate to the secondary school mathematics program in British Columbia. While a large majority is contained in Grades 11 and 12 mathematics courses, some are introduced in earlier grades. It is, further, noteworthy that the BC secondary mathematics program applies the *Strand* concept. In this curriculum model, educational concepts are re-visited in successive grades as the student learns a topic in greater depth and to enhanced extent. For this reason, an exact linking of Mathematical Proficiencies of the Survey and a Grade level is not possible, but a close approximation can be achieved.

Of the individual proficiencies of the Survey associated with the **Grade 12** or the **Grade 11** (including pre-Grade 11) mathematics courses, respondents rated, for *Calculus*, 60% of the Grade 12 topics and 74% of the Grade 11 topics as at least 3 on the 0 to 4 importance scale. The Steering Committee agreed that any Mathematical Proficiency having an importance rating of 2 or greater should be deemed sufficiently important to be regarded as a necessary entry proficiency to the applicable post-secondary course. The table (see page 17) summarises the importance rating proportions and the mean importance rating for all secondary school topics in Grade 12 or 11.

Sub-Proficiency Importance Rating for Calculus According to Grade Level

Secondary Mathematics Grade Level	Percent of Topics in Each Interval Importance Rating (IR) Scale: 4 to 0				Overall Mean Importance Rating
	$4 \geq IR \geq 3$	$3 > IR \geq 2$	$2 > IR \geq 1$	$1 > IR \geq 0$	
Grade 12	60	35	5	0	**3.07**
Grade 11*	74	16	0	10	**3.19**

***** This includes a few topics of pre-Grade 11 mathematics.

Notice that 95% and 90%, respectively, of the Grade 12 and Grade 11 mathematics topics (see page 48) were given an importance rating of 2 or greater for entry to *Calculus*. The responses to Part A of the questionnaire clearly demonstrate the very high importance that respondents attach to the topics in Grades 11 (3.19 rating) and Grade 12 (3.07 rating).

Some respondents also suggested specific proficiencies that they felt needed emphasis (see page 41). However, respondents appear generally content that secondary Grades 11 and 12 courses contain the specific topics expected of students in *Calculus*.

Introductory Statistics

All but one of the 24 British Columbia post-secondary educational institutions surveyed offer at least one *Introductory Statistics* course. Respondents were asked to consider an *Introductory Statistics* course to be one suitable for students having taken no previous, post-secondary, statistics course (see page 71). Further, such a course would not require, for entry, that a student have a background in calculus. It is noteworthy that, in some post-secondary educational programs, an *Introductory Statistics* course is a program requirement and that admission to such a course may depend upon having met program admission standards. Other *Introductory Statistics* courses set successful completion of secondary Grade 11 mathematics as the prerequisite.

When assessing the importance of the sub-proficiencies within the Proficiency Categories of the Survey, respondents rated 14% of individual proficiencies to be **Essential to Marginally Important** for *Introductory Statistics* with the majority (11%) falling in the **Quite Important to Marginally Important** range (see Appendix B, page 58). The proficiencies of *using and understanding the slope of a line*, *graphing and writing the equations for linear functions*, and *finding and writing up solutions to word problems* ranked at the top of the importance list for entering *Introductory Statistics* students.

Respondents rated only 17% of the 18 Proficiency Categories of the Survey as **Essential to Marginally Important** for *Introductory Statistics* (see page 19). It must be reiterated that this course typically has a Grade 11 prerequisite and that 6 of the 18 Categories describe proficiencies almost exclusively assigned to Grade 12 mathematics courses.

Proficiency Categories Ranking – INTRODUCTORY STATISTICS

Category Code	Category Description	Importance Rating	Mean of Sub-Proficiencies in Category
M	Understand and use Straight Line and Linear Functions.	**3.64**	**3.13**
J	Understand and use Data Analysis.	**2.34**	**2.21**
E	Solve Equations and Inequalities.	**2.30**	**0.88**
A	Understand and use Polynomial Expressions.	**1.95**	**1.10**
D	Understand and use Radical Expressions.	**1.89**	**1.03**
C	Understand and use Exponential Expressions.	**1.82**	**1.36**
L	Understand and use the Function Concept.	**1.77**	**1.22**
K	Understand and use Sequences and Series.	**1.55**	**1.21**
N	Understand and use the Quadratic Function.	**1.50**	**1.07**
B	Understand and use Rational Expressions.	**1.43**	**1.06**
R	Understand and use the Logarithmic Function.	**1.30**	**0.97**
F	Understand and use the Geometry of Lines and Points.	**1.23**	**0.49**
T	Understand and use some Concepts of the Calculus.	**0.86**	**0.59**
Q	Understand and use Polynomial Functions.	**0.68**	**0.28**
P	Understand and use Quadratic Relations.	**0.55**	**0.46**
H	Understand and use Triangle Trigonometry.	**0.45**	**0.28**
G	Understand and use the Geometry of Circles.	**0.32**	**0.20**
S	Understand and use Circular Trigonometric Functions.	**0.18**	**0.12**

The table above summarises, in column 3, the ranking of the Proficiency Categories for *Introductory Statistics* using the mean of the overall importance rating respondents gave to each Proficiency Category while column 4 shows the mean of the ratings for sub-proficiencies within each Proficiency Category.

The Steering Committee agreed that any Mathematical Proficiency having an importance rating of 2 or greater should be deemed sufficiently important to be regarded as a necessary entry proficiency for the applicable post-secondary course. That none of the Grade 12 topics are given a mean importance rating of 2 or greater is understandable. However, it is noteworthy that only **20%** of the **Grade 11 topics** are given an importance rating of 2 or greater. A better understanding of the mathematical proficiencies expected for this course is gained by examining the General Proficiencies for *Introductory Statistics* (see page 13) and by noting the suggested proficiencies (see page 42) valued by post-secondary statistics instructors.

Sub-Proficiency Importance Rating for Intro Stats According to Grade Level

Secondary Mathematics Grade Level	Percent of Topics in Each Interval Importance Rating (IR) Scale: 4 to 0				Overall Mean Importance Rating
	$4 \geq IR \geq 3$	$3 > IR \geq 2$	$2 > IR \geq 1$	$1 > IR \geq 0$	
Grade 12	0	0	19	81	**0.55**
Grade 11*	4	16	34	46	**1.26**

***** This includes a few topics of pre-Grade 11 mathematics.

When the individual, sub-proficiencies of the Survey with respect to *Introductory Statistics* are associated with the **Grade 12** or the **Grade 11** (including pre-Grade 11) mathematics courses, respondents gave a mean importance rating of 0.55, on the 0 to 4 scale, to topics from Grade 12, and only 1.26 to topics from Grade 11. The table above summarises the importance rating proportions indicated by the Survey for *Introductory Statistics*.

Respondents voiced frustration (see page 34) over the fact that some students in *Introductory Statistics* are taking the course as a required component of another program and that students failed to appreciate its relevance to their future. Also, it is obvious that, while respondents expect a degree of previous exposure to Data Analysis, students rarely show much proficiency or profess any recall of such experience. Although not made a formal recommendation, it was suggested by the Steering Committee that an item on Data Analysis could be included on a provincial final examination.

Lastly, the wide distribution of ratings from institution to institution given items on the *Introductory Statistics* questionnaire reveals a considerable range of expectations of students. The findings of the Survey, both on General and on Specific Proficiencies, should prove very useful in attempting to understand this variation.

The Necessity Principle: Students are most likely to learn when they see a need for what we intend to teach them, where by "need" is meant intellectual need, as opposed to social or economic need.

G. Harel, The American Mathematical Monthly, June-July, 1998, p. 501.

Mathematics for Elementary Education

> Virtually all young children like mathematics. They do mathematics naturally, discovering patterns and making conjectures based on observation. Natural curiosity is a powerful teacher, especially for mathematics.
>
> Unfortunately, as children become socialized by school and society, they begin to view mathematics as a rigid system of externally dictated rules governed by standards of accuracy, speed and memory. Their view of mathematics shifts gradually from enthusiasm to apprehension, from confidence to fear. Eventually, most students leave mathematics under duress, convinced that only geniuses can learn it. Later, as parents, they pass this conviction on to their children. Some even become teachers and convey this attitude to their students.
>
> Everybody Counts, p. 43.

Sixteen of the 24 post-secondary educational institutions surveyed reported offering a *Mathematics for Elementary Education* (MFEE) course or one transferring as such a course. Some institutions not offering an elementary education program themselves have negotiated transfer arrangements in which students, having for instance successfully completed a finite mathematics course, satisfy the elementary education mathematics requirements at another institution. A majority of MFEE courses admit students after Grade 11 mathematics, while most finite mathematics courses require mathematics at the Grade 12 level. In this Report, data from dedicated MFEE courses and courses transferring as such are not distinguished.

When assessing the importance of the sub-proficiencies within the Proficiency Categories of the Survey, respondents rated 15% of the individual proficiencies to be **Essential to Marginally Important** for MFEE (see Appendix B, page 60). The proficiencies of *solving linear equations and inequalities* and *finding and writing up solutions to word problems* ranked at the top of the importance list for entering *Mathematics for Elementary Education* students.

Of the 18 Proficiency Categories in the Survey, six of which describe proficiencies almost exclusively assigned to Grade 12 mathematics courses, respondents rated 39% of them as **Quite Important to Marginally Important** for entry to MFEE (see page 21). The marked difference (39% vs. 15%) in the proportions of the means lying in the 4 to 2 importance range, for Proficiency Categories as compared to the sub-proficiencies themselves, suggests a preference for general proficiencies as opposed to particular, individual competencies.

Proficiency Categories Ranking – MATHEMATICS For ELEMENTARY EDUCATION

Category Code	Category Description	Importance Rating	Mean of Sub-Proficiencies in Category
E	Solve Equations and Inequalities.	**2.94**	**1.10**
M	Understand and use Straight Line and Linear Functions.	**2.44**	**2.02**
A	Understand and use Polynomial Expressions.	**2.25**	**1.63**
C	Understand and use Exponential Expressions.	**2.25**	**1.61**
D	Understand and use Radical Expressions.	**2.25**	**1.48**
F	Understand and use the Geometry of Lines and Points.	**2.19**	**1.95**
B	Understand and use Rational Expressions.	**2.00**	**1.80**
L	Understand and use the Function Concept.	**1.88**	**1.11**
J	Understand and use Data Analysis.	**1.38**	**0.89**
G	Understand and use the Geometry of Circles.	**1.19**	**0.88**
N	Understand and use the Quadratic Function.	**1.13**	**0.97**
K	Understand and use Sequences and Series.	**1.00**	**0.70**
H	Understand and use Triangle Trigonometry.	**0.69**	**0.33**
R	Understand and use the Logarithmic Function.	**0.63**	**0.50**
P	Understand and use Quadratic Relations.	**0.50**	**0.26**
Q	Understand and use Polynomial Functions.	**0.44**	**0.11**
S	Understand and use Circular Trigonometric Functions.	**0.19**	**0.07**
T	Understand and use some Concepts of the Calculus.	**0.06**	**0.00**

The table above summarises, in column 3, the ranking of the Proficiency Categories for MFEE using the mean of the overall importance rating respondents gave to each Proficiency Category while column 4 shows the mean of the ratings for sub-proficiencies within each Proficiency Category. Even though Categories E and M were rated highly as proficiency groupings, it should be noted that all-embracing proficiencies such as the set concept (U5,U6) or the function concept (L1) ranked near the top of the individual proficiency list (see page 60).

As to the mathematical proficiencies of the Survey associated with the **Grade 12** or the **Grade 11** (including pre-Grade 11) mathematics courses, respondents rated none of the Grade 12 topics and only 26% of the Grade 11 topics as at least 2 on the 0 to 4 importance scale. The Steering Committee agreed that any Mathematical Proficiency having an importance rating of 2 or greater should be deemed sufficiently important to be regarded as a necessary entry proficiency to the applicable post-secondary course. The table (page 23) summarises the importance rating proportions and the mean importance rating for all secondary school topics in Grade 12 or 11 relative to MFEE.

Sub-Proficiency Importance Rating for MFEE According to Grade Level

Secondary Mathematics Grade Level	Percent of Topics in Each Interval Importance Rating (IR) Scale: 4 to 0				Overall Mean Importance Rating
	$4 \geq IR \geq 3$	$3 > IR \geq 2$	$2 > IR \geq 1$	$1 > IR \geq 0$	
Grade 12	0	0	13	87	**0.40**
Grade 11*	2	24	40	34	**1.43**

***** This includes a few topics of pre-Grade 11 mathematics.

Comments from individual respondents (see page 38) indicate that the mathematical proficiencies demonstrated by a large proportion of entering MFEE students at present compels instructors to incorporate a remedial component in their course. While stating a Grade 11 mathematics prerequisite for MFEE, instructors have lessened their specific proficiency expectations of entering students, which is reflected in the above table.

The considerable variability in proficiency expectations indicated by respondents to the MFEE questionnaire is a concern. As is the diversity of acceptable entry pathways to MFEE credit. Both should be addressed.

If the information gathered in this Survey, particularly that concerning general proficiency expectations for MFEE, is made generally available in an accessible, user-friendly form, then positive changes in the preparation for students to teach elementary mathematics should result.

A well documented observation in the cognitive science literature is that once a way of thinking is established as a behavior, it becomes tenacious and extremely difficult to relinquish. The implication of this finding is that the seeds for good ways of thinking must be laid early on in students' mathematical experience—in elementary and secondary education.

G. Harel, The American Mathematical Monthly, June-July, 1998, p. 506.

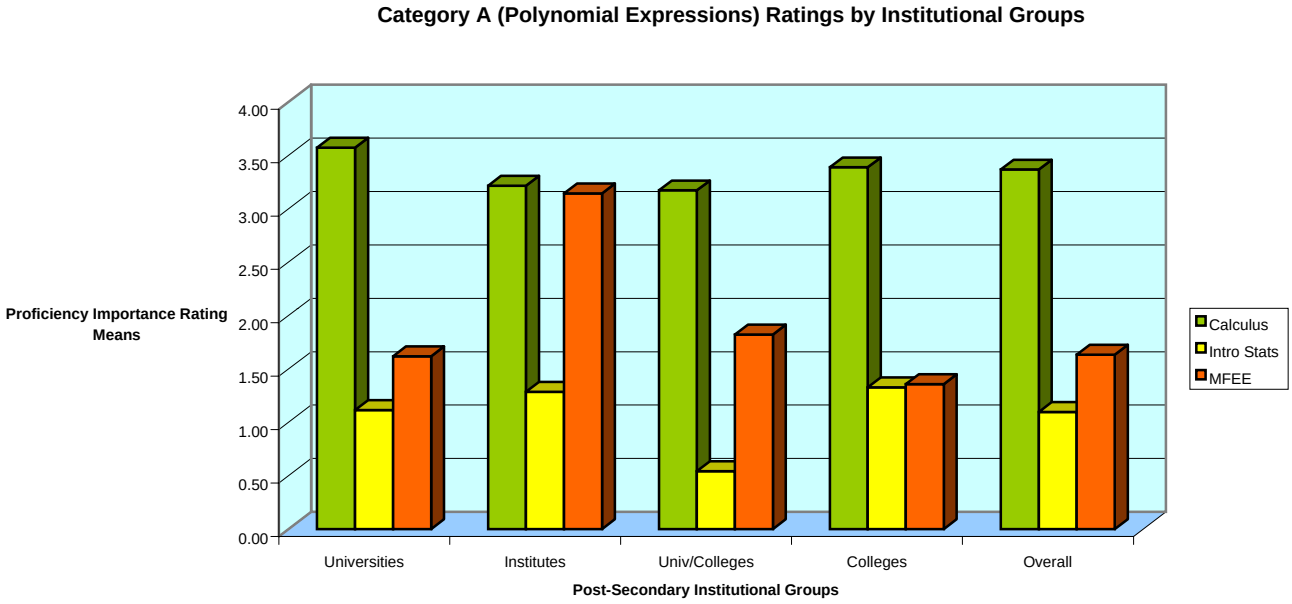
Survey Results - Institutional Groupings

As universities, university-colleges, institutes and community colleges began responding to the Proficiencies Survey of *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education*, the Project Steering Committee expressed interest in comparing some of the mathematical proficiencies of the Survey by post-secondary institutional group and according to the courses of the Survey. To this end, the post-secondary institutions involved in the Survey were grouped as shown in the **Institutional Groups** table (page 24):

Post-Secondary Institutional Groups

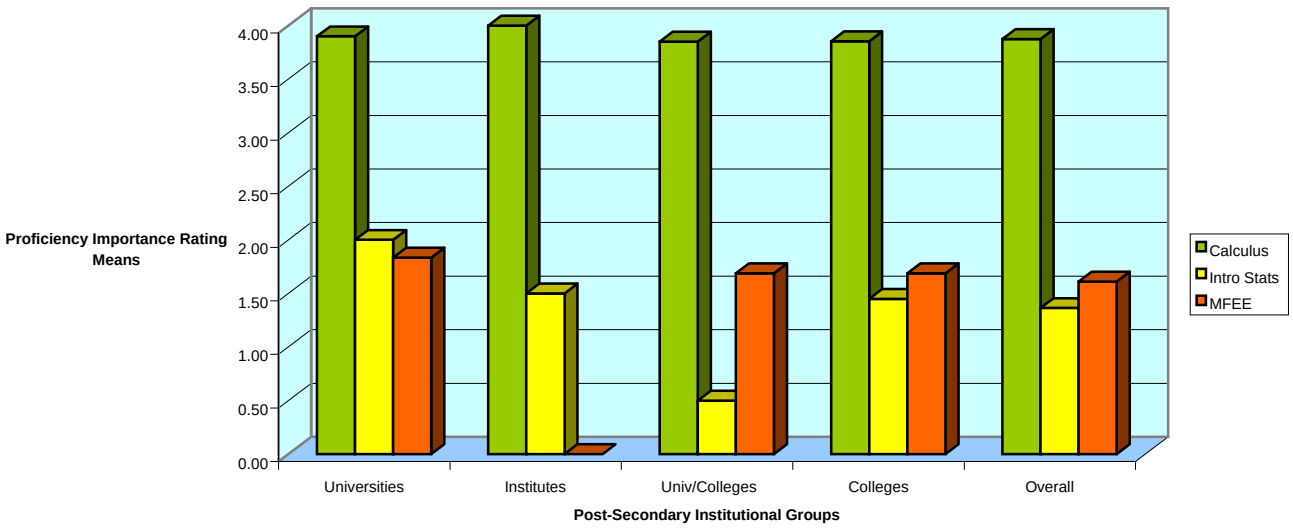
Universities	Institutes	University Colleges	Colleges
Simon Fraser University Trinity Western University The University of British Columbia University of Northern British Columbia University of Victoria	British Columbia Institute of Technology Open Learning Agency	Kwantlen University College Malaspina University-College Okanagan University College The University College of the Cariboo University College of the Fraser Valley	Camosun College Capilano College College of New Caledonia College of the Rockies Columbia College Coquitlam College Douglas College Langara College Northern Lights College North Island College Northwest Community College Selkirk College

Charts to display the mean importance ratings given by respondents to selected proficiency categories and depicted according to institutional grouping were constructed. Because respondents rated categories A, C, E and M among the six top Categories in each of the courses of the Survey, they were the ones chosen: A (Polynomial Expressions), C (Exponential Expressions), E (Equations and Inequalities) and M (Straight Line and Linear Functions). For each institutional grouping and each course of the Survey, the charts portray the mean importance rating for the sub-proficiencies of the specified Category.



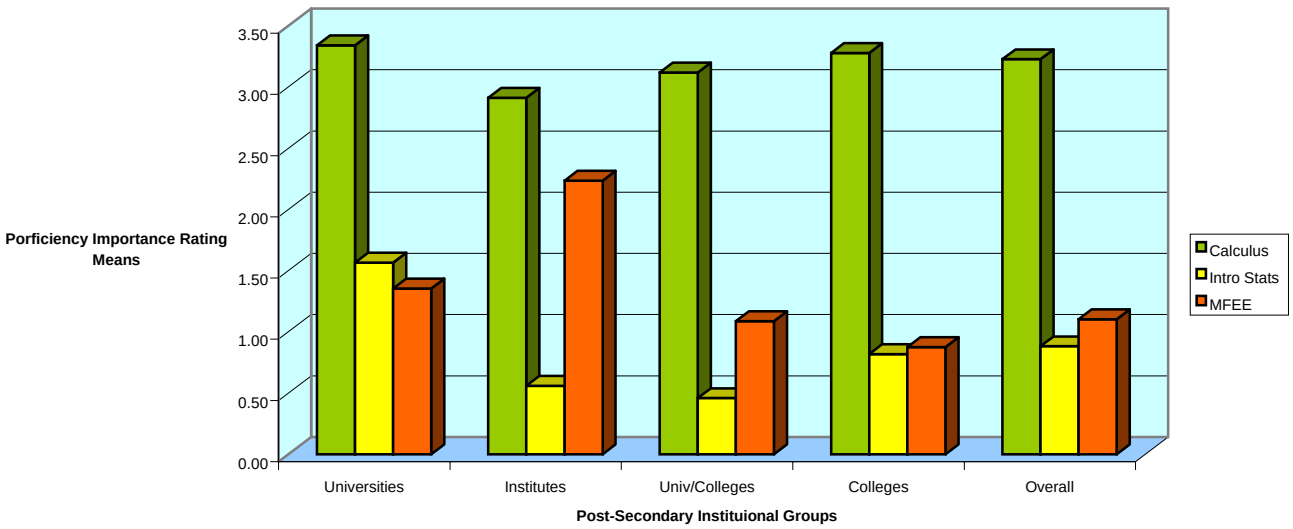
The very high importance rating given to *Calculus* in most Categories is evident.

Category C (Exponential Expressions) Ratings by Institutional Groups



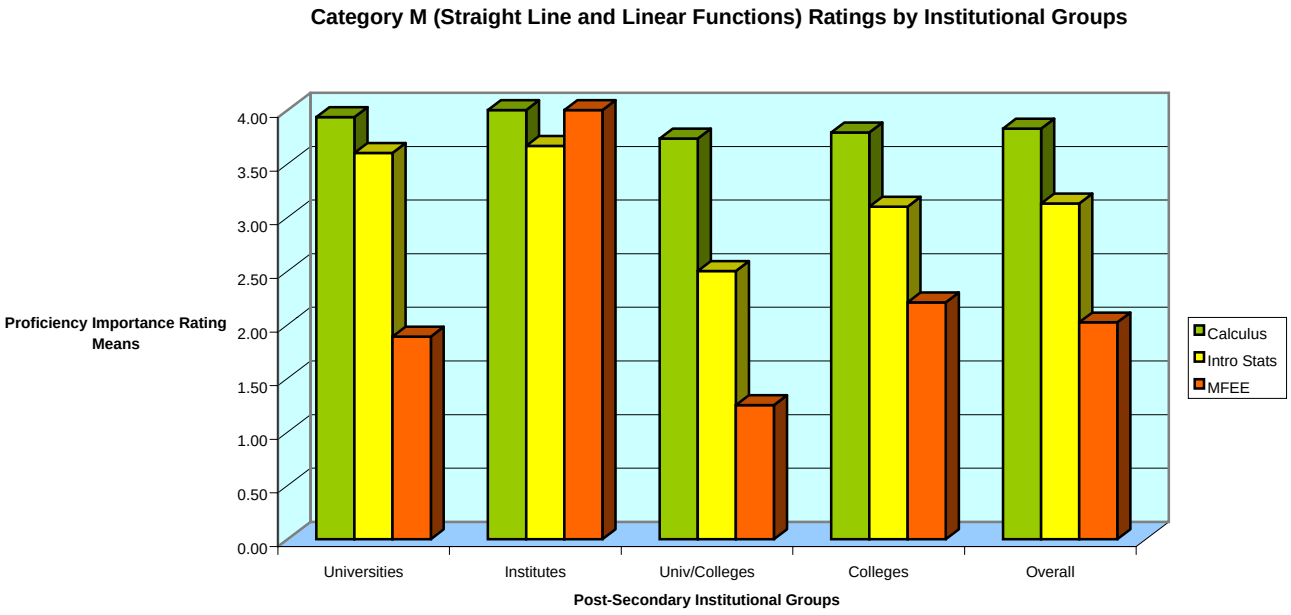
As an observation, when comparing the Category A with the Category C importance ratings, respondents rated the more complex skills of Category C slightly higher than those of Category A, which would typically be taught earlier.

Category E (Equations and Inequalities) Ratings by Institutional Groups



Introductory Statistics and MFEE respondents rate linear equation and inequality solving skills highly, but they do not require the entire repertoire of equation solving abilities in Category E. Category M is very interesting because it received the highest combined importance rating of all

eighteen Categories. This mathematical proficiency which unites the symbolic aspects of algebraic statements with the visual benefits of Cartesian graphs, thereby allowing both the cursory and the detailed analysis of results, is a pre-eminent proficiency for post-secondary mathematics/statistics courses.



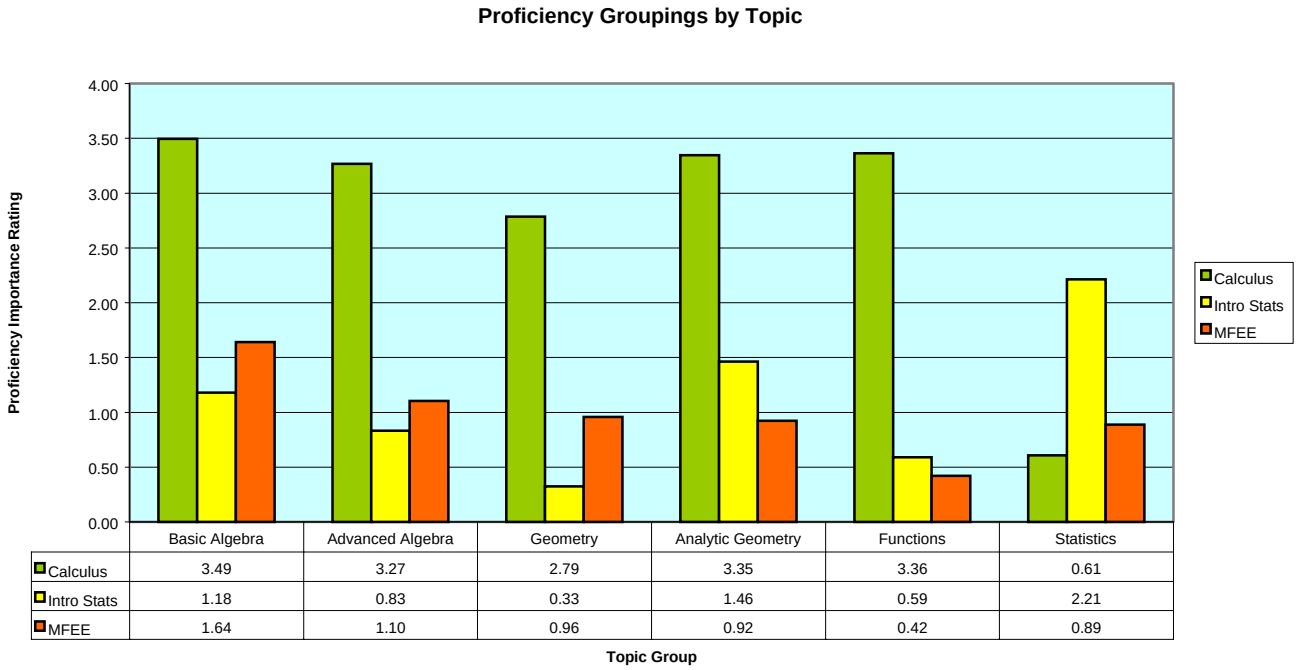
The importance rating responses for specific proficiencies vary from institution to institution. This same kind of variability is observed when institutions are grouped as was the case in the preceding charts. When compared with the Overall or combined rating for a particular Category, no great variation between post-secondary institutional groupings is noted. The **Institutes** group contains only two institutions and appears, for this reason, as would be expected, to vary most.

Survey Results – Grouping By Topic

Mathematics/Statistics instructors often use single labels to refer to clusters of topics or proficiencies. Although there is no established or rigid categorisation of mathematical topics, it is sometimes helpful to combine them. Post-secondary instructors prefer to discuss entry proficiencies, whether weak or strong, in broad terms. Further, when beginning the development or modification of pre-calculus courses in university or college, they have the custom of identifying large groups of topics before proceeding to the detail.

The secondary school curriculum for mathematics has also been organised into topic groupings called *Strands*. Besides forming a visible, curricular thread through the secondary mathematics program, the *Strand* system further enhances the monitoring and maintenance of developing themes in mathematics. Although the Mathematical Proficiencies of Part A of the Survey were not sorted according to *Strand*, it is possible to identify individual Proficiencies with them.

The chart below shows the importance ratings of proficiency groupings from Part A of the Survey. The Topic groups (with Proficiency Categories in parentheses) are: Basic Algebra (A,B,E), Advanced Algebra (C,D,E), Geometry (F,G,H), Analytic Geometry (M,P), Functions (L,N, Q,R,S) and Statistics (J).



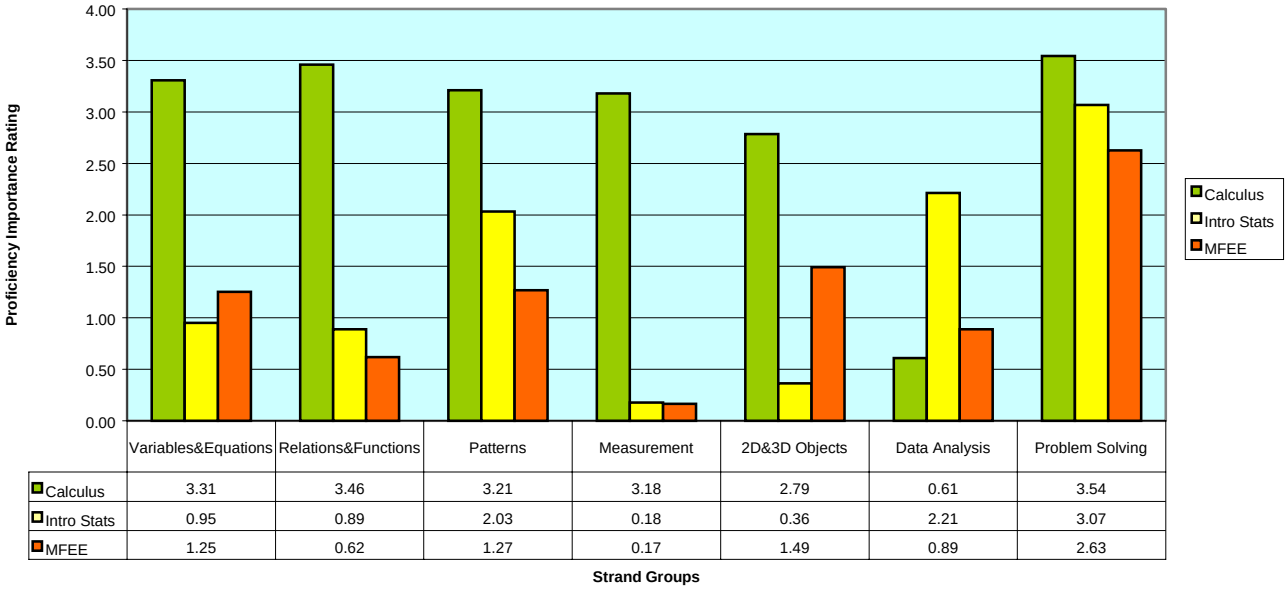
	Basic Algebra	Advanced Algebra	Geometry	Analytic Geometry	Functions	Statistics
Calculus	3.49	3.27	2.79	3.35	3.36	0.61
Intro Stats	1.18	0.83	0.33	1.46	0.59	2.21
MFEE	1.64	1.10	0.96	0.92	0.42	0.89

In grouping proficiencies by topic, it becomes clearer that statistics topics, for instance, are of little value for *Calculus* and that all topics are of some, but of very little, importance for *Mathematics for Elementary Education*.

As mentioned, the *Strand* system groups proficiencies differently from the Topic clustering used above. The *Strand* groupings (with Proficiency Categories in parentheses) for the chart on Page

28 are: Variables & Equations (A,B,C,D,E,N,Q), Relations & Functions (L,P,R), Patterns (K,M), Measurement (H,S), 2D & 3D Objects (F,G), Data Analysis (J) and Problem Solving (U1).

Proficiency Category Groupings by Strand



	Variables&Equations	Relations&Functions	Patterns	Measurement	2D&3D Objects	Data Analysis	Problem Solving
Calculus	3.31	3.46	3.21	3.18	2.79	0.61	3.54
Intro Stats	0.95	0.89	2.03	0.18	0.36	2.21	3.07
MFEE	1.25	0.62	1.27	0.17	1.49	0.89	2.63

It should be pointed out that the Measurement *Strand* at the Grade 11 and 12 level consists principally of trigonometry topics. Also, without diminishing its very high importance rating, the Problem Solving figures originate from the responses to line U1 of the Survey.

PART 2

ANECDOTAL and INDIVIDUAL FEEDBACK

Part B of the Mathematics Proficiencies Survey questionnaire offered respondents an opportunity for less structured feedback. Besides being asked to provide concrete examples of problems or exercises giving students difficulty, respondents also had freedom to comment on students' general preparedness for their post-secondary course and to offer suggestions concerning ways to improve their performance. **PART 2** of the Report outlines these responses.

CALCULUS – Comments and Suggestions

Here are some direct comments and helpful, specific suggestions respondents offered relative to students' general preparedness for *Calculus*:

- Most of the "common" difficulties I see arise, in my opinion, from an over-emphasis of recipe-learning rather than understanding strategies or approaches to problem solving. It's not so much that learning outcomes are missing in the students' backgrounds, but they've relied on unreliable approaches to learning.

- STUDENTS DO NOT HAVE THE ALGEBRA SKILLS REQUIRED TO SIMPLIFY MOST DERIVATIVES. They have knowledge of the skills and recognise what to do with some hints, but they lack skill in doing algebra.

- Taking an applied problem; the majority of students have very little experience and hence no skill in attacking problems. They should spend some considerable time studying and using Polya's problem solving approach.
  "e.g., A ladder 10 ft long rests against a vertical wall. Let θ be the angle between the top of the ladder and the wall, and let x be the distance from the bottom of the ladder to the wall. If the bottom of the ladder slides away from the wall, how fast does x change with respect to θ when $\theta = \frac{\pi}{3}$?"
  Most students would not even be able to begin to set up a solution to this problem and would never even get to the calculus part.

- Students should do more word problems after each section so they develop logical thinking!

- My experience has been that most students' single greatest weakness is in their understanding of circular trigonometric functions.

- I think that function modelling (building; interpreting; analysing; etc.) should serve as the central theme in the secondary school math curriculum.
  "a) Build a function model that expresses the power line construction cost as a function of *x*."
  - Most students don't know what the question is asking them to do.

  "b) In this application, what is the domain of the function in part a)?"
  - Applied (practical) domain as part of function model. Particularly important now as a part of the process for determining the window of graphing utility.

  "c) Which values of *x* allow you to construct the power line within a $2000 budget?"
  - Many students have trouble translating this as an inequality to be solved. Some (with previous calculus exposure) think that you must take derivative and set to 0 (rote application of a solution process typically applied to this type of question).

  Other comments: There should be greater emphasis placed on the geometric (numerical) solutions of equations and inequalities. The function concept (with complete functional notation, e.g., $f(x)$, $\ell n(x)$, $\sin(x)$, should be the central theme. The topics on linear, quadratic, logarithmic, exponential, trigonometric, absolute value and rational functions should be integrated with emphasis on model building. Where possible, the inverse function relationship should be introduced.

- Calculus students run into difficulty when studying anything in calculus relating to the following topics, simply because these topics are completely omitted or given "the-short-end-of-the-stick" in the Grade 12 math program:
  1. Inverse trigonometric functions.
  2. Absolute value functions and their graphs.
  3. Implicit functions.
  4. Partial fractions.
  5. Solving inequalities involving absolute value functions.
  6. Graphic $+,-,\times,\div$ of functions, along with a graphic approach to composite functions.
  7. Graphic solution of triangle problems.
  8. Descriptive geometry.
  9. The number *e*: its origin, e^x and $\ell n(x)$.
  10. Parametric equations.
  11. Complex Numbers.

- At the college level students often need to be able to juggle two or more "pieces" of information, and I have a sense that often this is mathematical practice with which they have little experience. For example, the definition of absolute value with which we work requires them to use a particular formula in one case but a different formula in another case—they need to manage both the condition and the formula. Another pre-calculus topic where such juggling is necessary is the definition of the domain of the composition of two functions—they need to think about those inputs for *g* where the values of *g* will be inputs for *f* and my experience suggests they struggle terribly when trying to manage all those conditions. I am not convinced that the formal definition of absolute value or of the domain of function compositions are particularly critical to a study of calculus. Even the most basic concepts of calculus (such as rate of change or limits) involve managing complex "collections" of information.

  Another practice that seems to be missed by focussing on topics of the curriculum is mathematicians' propensity for abstracting and generalising. My own research on students' understanding of change

and rate of change suggested that students can quite readily get their heads around these notions geometrically and numerically but have difficulty with algebraic abstractions, such as $\Delta f/\Delta x$, that describe them.

- In examining the final exams of students who failed my 1st semester calculus course, two deficiencies stand out:
  1) Function Construction – My final exams always contain an applied max/min problem and a related rate problem. Of those students who failed the final exam, none correctly constructed the necessary function needed to answer the problem.
  2) Multi-step Problems – Students who fail 1st semester calculus typically cannot successfully complete multi-step questions; although they may be able to answer correctly the individual parts of the question if these were asked separately (see page 33).

These quotations convey a deep concern for student success in *Calculus* and a sense of frustration about the lack of mathematical proficiency on the part of some. They underscore a common message concerning the **general proficiencies** (see page 13) that post-secondary instructors expect of students entering their *Calculus* course.

It is acknowledged that general proficiencies are difficult, if not impossible, to develop without regard to specific skills and competencies as well. However, a student with the ability to respond quickly to short answer questions and to recall important facts correctly, may be unable to link any of these proficiencies so as to solve a typical, multi-step problem. Students may also be unaware of the importance of unifying themes, such as the function concept or the set concept, in mathematics. If such deeper understandings are nurtured throughout the study of secondary mathematics, students gain not only a better appreciation for mathematical coherence, but also a more-comprehensive base on which to built a further study of mathematics.

The Mathematics Proficiencies Steering Committee urged that mathematics examinations for secondary students contain questions requiring multiple-step solutions. It was acknowledged that such a practice would reduce the number of individual questions that could reasonably be asked on examinations. However, the need for students to be further along in their development of general proficiencies, such as thinking and analysis abilities, familiarity with unifying themes in mathematics and problem modelling skills, when they arrive in *Calculus*, was repeatedly expressed. Without these general proficiencies to provide context and meaning, students soon lose specific mathematical skills leaving them to appear weak even in basic algebra skills.

CALCULUS – Problems of Interest to Calculus Students

The Survey (Part B) also requested that respondents cite examples of specific problems or questions that, in their experience, give students difficulty for reasons of poor background or mathematical proficiencies. They were then asked to identify, where appropriate, the particular deficiency that underlay the perceived weakness.

Some respondents accepted the challenge and provided the following problems. Some of the problems represent those that students entering *Calculus* should handle with ease, while the second category contains those with which students unnecessarily struggle in a *Calculus* course.

Examples of Pre-Calculus Problems

Three examples of "common" misunderstandings:

1. Correctly using and sorting out percents.

"Hank now earns $30 000 per year after a 7.5% pay raise. What was his salary last year?"

2. Unit conversion and a sense of what is a reasonable answer.

"Given that 1 cm^3 H_2O has a mass of 1g, and 1 lb = 454 g, 1 in = 2.54 cm, 1 ft = 12 in, find the weight in lb of 1 ft^3 of H_2O."

Perhaps 9 out of 10 students will come up with an answer something like 0.0006 lb/ft^3—wrong and no sense that this is ridiculous.

3. Cancellation of common factors in fractions—all sorts of variations on this problem.

"$\frac{x+5}{x-3}=\frac{x-4}{x+5}$" – cancelling across equal signs.

"$\frac{x+5-x^2}{x+5+x^3}$" – not common factors.

"If x is within a distance d of 2, then how far is x^2 from 4?"

Proficiency lacked—a sense of absolute value and inequality.

Factor and simplify:

"$\dfrac{3x^2(2x-1)^{\frac{1}{2}}-x^3\left(\frac{1}{2}\right)(2x-1)^{\frac{-1}{2}}\cdot 2}{\left[(2x-1)^{\frac{1}{2}}\right]^2}$" or "$x^{\frac{2}{3}}+x^{\frac{-1}{3}}$"

Difficulty in factoring expressions with rational exponents.

Very Basic! "On the given grid, draw a line with slope 4 passing through the point (-3,3)."

Examples of Problems Students Encounter in a Calculus Course

Respondents offered the following problems as ones with which students have difficulty for reasons of inadequate mathematical proficiencies:

"Find the derivative of $g(t)=\left(\dfrac{t-2}{2t+1}\right)^9$. Simplify the answer."

Suppose the yearly Gross Domestic Product (GDP) of a small country is presently $80 billion and is increasing at an instantaneous rate of $2 billion per year. Furthermore, assume that the population of the country is presently 4 million people and is increasing at an instantaneous rate of 0.4 million people per year. If the per capita GDP is defined as the GDP per person, then what is the present instantaneous rate of change of the per capita GDP of the country?

"A water tank has the shape of an inverted circular cone with base radius 2 m and height 4 m. If water is being pumped into the tank at a rate of 2 m^3/ min, find the rate at which the water level is rising when the water is 3 m deep."
The proficiency needed: understand and apply the principle of similar triangles.

You are in a tank (military variety) moving down the y-axis towards the origin. At time $t = 0$, you are 3 km from the origin; 10 minutes later you are 2 km from the origin. Your speed is decreasing; it is proportional to your distance from the origin.

You know that an enemy tank is located somewhere on the positive x-axis. Unfortunately, a high wall along the curve $xy = 1$ prevents you from seeing exactly where it is. How fast must your gun turret be capable of turning to maximise your chances of surviving the encounter?

"Find the coordinates of the two points of the graph of $y = x^2$ so that tangent lines to the graph also contain the point (a,b)."

Here is an illustrated example of a multi-step problem:

Find the interval on which $f(x) = \dfrac{x^2+9}{x}$ is increasing.

a) $f'(x) = \dfrac{2x(x)-(x^2+9)}{x^2}$

b) Simplify: $\dfrac{2x(x)-(x^2+9)}{x^2} = \dfrac{x^2-9}{x^2}$

c) Factor the numerator: $\dfrac{x^2-9}{x^2} = \dfrac{(x+3)(x-3)}{x^2}$

d) Solve: $\dfrac{(x+3)(x-3)}{x^2} > 0$, $x < -3$ or $x > 3$

e) What condition is required on $f'(x)$ for $f(x)$ to be increasing? $f'(x) > 0$

f) Find the intervals on which $f(x)$ is increasing. $x < -3$ or $x > 3$

INTRODUCTORY STATISTICS – Comments and Suggestions

What follows are direct quotations from the responses to the *Introductory Statistics* questionnaire concerning the expectations of students entering that course:

- [Reading English and Writing English] - vital skills.
  Students are asked to read and critique newspaper accounts of statistical surveys and studies.

- The biggest problem is that of getting used to the vocabulary or jargon and learning to express oneself accurately. But, these are verbal proficiencies.

- A general obstacle for teaching/learning Statistics is that many students have heard too many times about the myth that "probability/statistics is hard". Some students, especially some social sciences students, enter their compulsory Introductory Statistics course with a lot of anxiety. A survey that I did among Stat 103 students at SFU some years ago indicated that a majority of the students would never have taken a statistics course if they did not have to, because they believed that Statistics is hard. I think that, in high school, along with the introduction of some basic concepts and simple application of Probability and Statistics, more stress of the importance and relevance of Statistics in our daily life and a correction of the myth before the students have heard it would be helpful.

- The Data Analysis category contains the main topics in a typical, post-secondary Intro Stats course, so while prior exposure to them may be helpful they cannot be considered as proficiencies of students entering the course.

  Some previous exposure to sequences is helpful since probability typically is defined as the limit of a sequence.

  I had considerable difficulty identifying the particular proficiencies from the set presented in this questionnaire. For students to succeed in intro stats they must be able to think critically, analytically and be familiar in a general sense with mathematical symbolism (and basic algebra and set operations). Students who have difficulty typically perceive mathematics as a disjoint collection of formulas and manipulative operations, and have a great deal of difficulty relating any of the topics to "real life".

- 1. Our studies indicate that English mastery is a better predictor of success in Intro Stats. However, students that arrive with a fear of numbers, fear of calculators, and less than Grade 11 Math at some previous time are also likely to be unsuccessful in Stats.
  2. Some students can learn to do the fairly minimal algebraic and arithmetic manipulations during the course.
  3. The above ratings are for the social sciences stats course (which is university transferable). The UT business stats course would have very different ratings.
  4. Our Intro Stats courses focus on interpretation of computer printouts, and on analysis of existing statistical studies. Students are required to do a minimum of number crunching by hand—the course is very verbal. However, those with no confidence about numbers are rarely

able to get past the calculations. Therefore, the current prerequisite is Math 11, as a measure of some level of comfort with mathematical ideas.

- The algebra skills of our students are quite varied. Our Math 160 (Intro Stats) course has a prerequisite of a C grade in Math 12; however, often students are permitted to take the course with lower grades and are not necessarily at a disadvantage. One common shortcoming of students (regardless of prerequisite) is their problem solving/logic skills. Whether calculating probabilities or tests (t, X^2, z, . . .), students seem to have a problem with organising/visualising the information in a question to then put into the appropriate calculation.
  Ex. *i*) z vs. t test *ii*) combination vs. permutation *iii*) conditional probability from a Venn diagram.

- I generally find that students who have taken "confidence interval" in school really only formula crunch and do not really understand the meaning of a confidence interval. I would prefer high schools to restrict themselves to J1 and J2, and perhaps play with odds and probability.

  Generally students who have taken Math 12 within the last 5 years with or without any statistical/probability components are adequately prepared for an introductory, non-calculus-based stats course.

  If students were somewhat proficient in Excel w.r.t. data analysis, it would be helpful. Going beyond data analysis into inferential statistics is of lesser value given the time constraints on the high school curriculum.

- The specific area students have most difficulty in is probability theory. This relates to their difficulty in distinguishing among combinations, permutations, partitions, etc. Tied to this is a fear/dislike of word problems.

- Students have to understand SD = $\sqrt{\frac{\sum (x-\bar{x})^2}{(n-1)}}$ and so on. But again, no algebraic manipulation. That would be done on $(SD)^2$ rather than on SD.

  On calculus, if students have the calculus background, then it can be used in teaching to discuss minimised sums of squares, etc., e.g., slope is zero at max. or min.

- Understanding of the Central Limit Theorem:
  –can't distinguish between probability distribution and the data histogram, eg., think large sample makes the population normal.
  –can't distinguish between sample and population, eg., think large population makes population normal.
  Proficiencies J and U11 (computer demo of CLT) should help.
  Understanding of the purpose of sampling:
  –often mistake parameters as statistics, and vice versa. Proficiency U11 (demo of parameter estimation) should help. [Respondent added U11 – Understand/interpret computer output.]

- At the college level, there are two distinct groups that will be taking an introductory stats course: the university transfer group and the career group. Each group has its own prerequisite demands. It is assumed that the U/T group has had some stats prior to entry into the stats course. I have received the Integrated Resource Package (IRP) from the ministry of Education and it appears that school students begin the study of stats in Grade 8 and continue to cover stats material up to and including Grade 11. Unfortunately, looking at the course outlines and the sample problems provided in the IRP, I'm not sure what the focus is in the topic of stats. It appears that an attempt is made to introduce the students to both the descriptive and inferential sides of statistics. There does not seem to be much difference in the course content for both the applications math stream or the principles math stream.

  Having taught both the U/T and careers stream, I found that the students in both groups had little or no understanding of the stats they covered in school. It was necessary for me to go right back to very basic material in stats. Otherwise, the students were lost right off the bat and never recouped.

  If I had my druthers, I would like to see a proper job done in the school system of the following topics:

  set theory, Venn diagrams, calculating empirical probabilities, etc, counting techniques, permutations and combinations, tree diagrams, descriptive stats and all that entails.

  If these topics were well-covered in school, then these topics could be reviewed quickly in college and U/T introductory stats courses, then a more in-depth coverage of inferential statistics could be made.

- Introductory Stats students (non-calculus based) have problems with algebra as simple as:

$$\frac{\sqrt{pqn}}{n} = \sqrt{\frac{pq}{n}}$$

  i.e., need proficiency: $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

As with *Calculus*, respondents to the *Introductory Statistics* Survey (Part B) voiced a profound desire for entering students to display **general proficiencies**. They point to facility with English, thinking and logical skills, abilities to handle lengthy word problems and experience in understanding/interpreting computer results as some of the vital, general proficiencies required for success in *Introductory Statistics*. It is also acknowledged that "some students can learn to do the fairly minimal algebraic and arithmetic manipulations during the course," but that lacking these general skills (see page 13) students are certain to struggle and, often, to fail.

Although specific mathematical competencies are needed for *Introductory Statistics* courses, respondents are somewhat divided on which (from Part A of the Survey) are most important (see Appendix B). Variability from institution to institution in the entrance criteria, which are often set by external programs for these courses, is one reason for a lack of consensus concerning a core skill set *for Introductory Statistics*. However, agreement exists on the importance of general proficiencies such as comfort in solving word problems and familiarity with mathematical symbolism.

When comparing general mathematical proficiencies, it is also noteworthy that *Calculus* respondents emphasise mathematical modelling and solving multi-step problems, while *Introductory Statistics* respondents call for reading/writing competencies and for logic/thinking skills. However, both groups stress the importance of problem solving abilities, particularly in the context of word problems.

Examples of Pre-Introductory Statistics Problems

Respondents to Part B of the *Introductory Statistics* Survey offered the following illustrations as examples of pre-statistics settings giving those students with a low mathematics proficiency level significant difficulty. Although these are individual illustrations and not intended to span the spectrum of perceived weaknesses, they do provide an informal glimpse of the level and kinds of expectations post-secondary instructors have of entering students:

* Order of operations: "$y = 14 - 6(2)$" ; "$14 - 6x \neq 8x$"

* Solve for x: "$2.4 = \frac{x - 50}{5}$"

"A baseball league has 4 divisions: A, B, C, D. A and B both have 6 teams; C and D have 7 teams. David guesses a winning team from each of the 4 divisions. What is the probability that at least one of his guesses is correct?"

Examples of Problems Students Encounter in an Introductory Statistics Course

Introductory Statistics students, as do most students in post-secondary mathematics/statistics courses, encounter problem exercises as a routine component of their learning experience. The following examples submitted by respondents are illustrative of such problems, and strongly underscore the importance of students' comfort and proficiency in handling data, using mathematical symbolism and with some fundamental notions of secondary mathematics:

Given n pairs of data $(x_1, y_1), (x_2, y_2), \ldots (x_n, y_n)$, write the equation of the best-fitting (regression) line. Some students have difficulty with the proficiencies in Category M (Understand and use Straight Line and Linear Functions).

"Problem 1: A small drugstore orders copies of a news magazine for its magazine rack each week. Let X= demand for the magazine, with probability distribution of:

X	1	2	3	4	5	6
P(X)	1/15	2/15	3/15	4/15	3/15	2/15

Suppose the store owner actually pays \$0.25 for each copy of the news magazine and the price to customers is \$1.00. If the magazines left at the end of the week have no salvage value, is it better to order 3 or 4 copies of the magazine?"

Most students know that they need to compare the expected net revenues when the order is 3 or 4 copies of

the magazine, but the difficulty is to come up with the function of net revenue h(X).
If ordering 3 copies, h(X) = X – 0.75 for X = 1,2, and h(X) = 2.25 for X = 3,4,5,6.
If ordering 4 copies, h(X) = X – 1.0 for X = 1,2,3 and h(X) = 3.0 for X = 4,5,6. Computing E(h(X)) and comparing its values is not a problem once h(X) is given. I believe that a strong background of finding and writing up solutions (U1) would help. More specifically, a strong background in translating word problems into math equations should be helpful.

"Problem 2: A result called Chebyshev's Inequality states that for any probability distribution of a random variable X and any number *k* that is at least 1, P($|X - \mu| \geq k\sigma$) $\leq 1/k^2$. What is the value the upper bound of *k*=2?"
Some students have difficulty with this problem because of a lack of background in solving inequalities with absolute value expressions (E18).

MATHEMATICS FOR ELEMENTARY EDUCATION – Comments and Suggestions

The following quotations from Part B of the Mathematics for Elementary Education Survey characterise the perceptions of respondents concerning the mathematical proficiency status of entering students and provide a clear sense of the **general proficiencies** that post-secondary instructors expect of them:

- Logic often proves a stumbling block. More set theory would help.
- Students enter this program with great variations in both ability and knowledge. However, since, in the course, most of the basic mathematical concepts are (re-)presented, ability and effort are much more likely to predict success than any previous knowledge of the skills outlined in this survey.
- Students come from a wide variety of backgrounds in this course. One area that seems to be quite weak generally is working with fractions (adding, subtracting, multiplying, dividing, simplifying). Another is percentages and ratios.
- In Math for Elementary Education, just about any single deficiency can be overcome.
- A number of proficiencies that I rated with a high score are actually things we teach in the course. If a student is missing one or two, they can learn them as we go, but, if they are missing too many, they are doomed. If students did have all of these proficiencies, we could have a better course—one that focuses on how these should be taught instead of on the content itself.
- Let's face it. The real agenda for this course is to overcome the fear and hatred of mathematics instilled in these (nearly all) women back when they were in grade two—so they do not perpetuate it. The course needs to root mathematics in the (to them more-friendly) cultural, historical and linguistic context. Frankly, the more-technical topics are unimportant to people who start to sweat when they see a numeral written down, yet who aspire themselves to teach primary school and shape the attitudes of another generation.

Respondents appear to voice resignation concerning the status of mathematical proficiency

possessed by entering *Mathematics for Elementary Education* (MFEE) students. Instructors of MFEE courses and of ones transferring as such seem committed to making some improvement in their students' attitude towards mathematics, and in raising their competence and confidence levels in certain areas of mathematics. However, the strong insistence, as is the case with *Calculus* respondents, that entering students present a well-defined set of mathematical proficiencies seems to be lacking for this course. MFEE respondents mention the general proficiencies of a positive attitude towards mathematics and of greater competence with some, fundamental mathematical concepts or procedures as desired qualities in their students. But, a general feeling of submission pervades these comments.

Because of the vital importance of MFEE to the maintenance of a vibrant mathematics program in our school system, the Mathematics Proficiencies Steering Committee felt that a review of not only the proficiency expectations for entering MFEE students, but also of the various standards applied to these courses, should be considered.

Conclusions, Some Characteristics of Eighth Grade Mathematics Classes in the TIMSS Videotape Study.

The most striking finding in this review of 90 classes was the rarity of explicit mathematical reasoning in the classes. The almost total absence of explicit mathematical reasoning in Algebra and Before Algebra courses raises serious questions about the ways in which those subjects are taught. In order for these courses to help introduce students to mathematical ways of knowing, some of the logical foundations of mathematical knowledge should be explicit.

The reform movement appears to support increasing the proportion of problems that are solver controlled, multi-step, and concrete. The Japanese classes in this study had significantly greater proportions of classes with predominantly solver controlled and multi-step problems.

A.B. Manaster, The American Mathematical Monthly, Nov. 1998, p. 803, 804.

Examples of Problems Students Encounter in an MFEE Course

Respondents offering courses transferring as MFEE courses gave the following examples of problems which students find difficult in these courses:

1. Word Problems: Two or more variables.

You have $100 000 to invest in stocks, bonds and the money market. Stocks give a 12% return, bonds give an 8% return and the money market gives a 4% return. Money invested in stocks should equal the sum of money invested in bonds and in the money market. The amount of money invested in stocks should be 3 times the amount invested in the money market. How will the money be allocated?

2. Counting Techniques

In order to obtain a driver's license, four different driving tests are given. One of the four is selected for each applicant. If a group of 2 women and 3 men apply for a driver's license, what is the probability that exactly 2 of the 5 will take the same test?

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1) Graph the feasible set represented by *n* inequalities. Be sure to label each corner.

$$-3x + y \leq 2$$
$$-2x + y \leq -1$$
$$x + y \leq 7$$
$$x, y \geq 0$$

Proficiencies: Solving systems of linear equations.
Graphing systems of inequalities.

2) An urn contains 3 white balls, 2 red balls and 4 black balls. What is the probability that a random three ball sample will contain at least one black ball?

Proficiencies: Combinations, factorials, solving word problems.

Myth: *Success in mathematics depends more on innate ability than on hard work.*

Reality: Sustained effort can carry most students to a satisfactory level of achievement in mathematics. Compare music and mathematics: although in both areas genetic factors clearly play a role at the very highest levels of creative achievement, parents and teachers generally believe that children can learn to play music at a reasonable level if only they exert sufficient effort. As a consequence, many students achieve success and personal satisfaction from their study of music. Whenever parents or teachers believe that genetic ability is the primary factor contributing to success in mathematics, students are likely to fail before they begin; when expectations of success are high, so is the resulting performance.

Moving Beyond Myths, p. 10.

PROFICIENCIES SUGGESTED BY RESPONDENTS

Respondents to the Survey (Part A) were encouraged to "write in" specific mathematical proficiencies that were, in their view, overlooked or did not receive sufficient emphasis. Some of these respondents also gave such additional proficiencies an importance rating. Besides helping to complete the profile of competencies expected for entry to post-secondary mathematics/statistics courses, these additional proficiencies will also be an invaluable resource to future studies of other first-year courses or for refinements of the present one.

There are thirty-nine mathematical topics suggested in the following tables:

For Calculus

Category	Suggested Proficiency	Comment	Number of Respondents
A	Divide polynomials with binomials of degree greater than 1, using "long division".		1
B	Writing a rational expression as partial fractions.		1
C	Factor expressions with rational exponents.		1
C	Expand $(a+b)^n$ where *n* is a positive integer.	Useful in proving the Power Rule.	1
D	Graphing radical functions.		1
E	Solve quadratic equations by factoring.		1
E	Solve inequalities containing absolute value expressions.		1
E	Solve equations and inequalities using numerical methods.		1
F	Understand and apply the properties of similar triangles.		2
H	Multiplication of a vector by a scalar.		1
L	Handling piece-wise functions.		1
L	Determine the domain and range of a function.	Various domains, as in applications.	4
L	Introduce explicit and implicit forms for functions and relations.		1
L	Analysing features of functions such as symmetry and asymptotes—rational functions.	Includes solving inequalities.	3
M	Graph and write linear inequalities.		1
M	Understand slope as constant rate.		1
N	Graph and write quadratic inequalities.		1
Q	Exploration of basic shapes of cubics and quartics.	Graphing utility.	1
Q	Bounds for the zeros of a polynomial function.		1
R	Emphasise inverse function relationship—log and exponential.		3
S	Polar $\leftrightarrow$ Rectangular conversion.		2
S	Introduction of inverse trigonometric functions.		2
T	Limits in the context of rational functions.		1
U	Introduction to parametric form.	Motion applications.	1

Those proficiencies identified for additional mention by more than one respondent are especially noteworthy. In some cases, the proficiencies relate to topics in the secondary school curriculum and, in others, they voice an expectation that could only be realised if these topics became part of pre-calculus programs.

It is very important to notice that, as a group, these additional proficiencies stress an emphasis on visualisation (graphing) and an increased use of the function concept as a theme for pre-calculus. They further urge the extension of 'proficiency' topics through the use of contextual problems and by increasing the range of related learning opportunities or applications. This latter expectation is seen, for example, in B, H and Q in the above table.

For Introductory Statistics

Category	Suggested Proficiency	Comment	Number of Respondents
C	Understand e^x.	As in $e^{-\lambda t}(\lambda t)^x / x!$	1
D	Understand what a square root is!		1
E	Solve inequalities containing absolute value expressions.		1
J	Interpret graphical displays of data.		1
L	Using the jargon of functions.	Example, *mapping*.	1
M	Write the equation of the regression line.		2
M	Be able to graph points (x_i, y_i).	Assumed in M1, but this notation should be used.	1
M	Be able to graph $y = \beta_0 + \beta_1 x$ and interpret the constants.	M1	1
N	Plot quadratic expressions without finding equations.	Sums of squares of deviations as a function of parameter values.	1
R	Apply logarithmic transformations to data.		1
T	Understand area under a curve.		1
U	Understand and interpret computer output.		1

The overall expectations for *Introductory Statistics* from respondents of the survey concern such global proficiencies as understanding radical notation, being conversant with the concept of function, and having sufficient comprehension of linearity and variation to interpret graphical data. The respondents state that manipulative skill is of much lower priority than is an overall understanding of mathematical concepts. They point out that computational devices are commonly used in *Introductory Statistics* classes and that students with a strong capability of interpreting calculator or computer outputs have a decided edge.

For Mathematics for Elementary Education

Category	Suggested Proficiency	Comment	Number of Respondents
C	Understand and use the Binomial Theorem.		1
F	Understand the terms of basic descriptive geometry and the properties of figures.		1
U	Use and understand combinatorics (permutations, combinations).		1

The number (16 out of 24) of post-secondary institutions in British Columbia offering a *Mathematics for Elementary Education* course is smaller than those offering either *Calculus* or *Introductory Statistics*. This and the fact that the mathematical proficiencies expected for MFEE were more than adequately covered in the Survey resulted in only 3 additional topics.

Respondents, under general comments in the Survey, repeated the desire that entering students have a larger set of basic, mathematical competencies than is now the case. They felt that weaknesses in some basic skills could be overcome, but that those students with little skill in and understanding of, for example, the four basic operations and their algorithms, are not likely to succeed.

The Proficiency-based Admission Standards System (PASS) has identified seven proficiencies in Mathematics that students will need to demonstrate for admission to Oregon University System institutions beginning in the fall of 2001:

- Perform Algebraic Operations
- Use Functions to Understand Mathematical Relationships
- Use Geometric Concepts and Models
- Use Probability and Statistics to Collect and Study Data
- Estimate and Compute
- Solve Mathematical Problems
- Reason Mathematically

http://pass-ous.uoregon.edu/standards/math/math_students.html

SUMMARY

The Mathematics Proficiencies Project set about to explore the feasibility of describing, with greater detail than is presently the case, the mathematical proficiencies required for entry to post-secondary *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education*. To achieve this aim, twenty-four educational institutions in BC were surveyed in regard to their mathematical expectations of students entering these courses.

Both the Steering Committee and respondents to the Survey seemed generally content with the proficiencies listed in the questionnaire. Some respondents expressed unhappiness over particular weaknesses in specific skills displayed by their students. Where there had been insufficient emphasis or an omission of proficiencies in the questionnaire, they suggested additional items for the list of proficiencies.

However, the message received from all sources was that students should have a strong set of general proficiencies when entering post-secondary mathematics/statistics courses. They should be comfortable with their general grasp of mathematics and their level of mathematical ability. More specifically, competencies in multi-step problem solving, logic and thinking skills, and fluency in the use of language are but a few of these generic proficiencies which are vital to the success of any student. Ways to improve the general proficiencies of entering students must be adopted.

The Steering Committee has proposed several recommendations that could lead to an improvement in student preparedness for post-secondary mathematics/statistics. It has urged that dialogue among the stakeholders involved be initiated or strengthened in order to establish educational strategies for optimising student readiness for further education. So that greater, general awareness of proficiency expectations can become a reality, the Committee also recommends that proficiency information in user-friendly form be made widely available.

The findings of this study and the recommendations, if implemented, should have a significant impact on the following:

- Motivation of students preparing for post-secondary studies.
- Emphases placed on the various topics in the secondary mathematics curriculum.
- Discussions about changes to this curriculum and the way students are evaluated.
- Quality of the communication between secondary and post-secondary educators.
- Reliability of the expectations held by post-secondary instructors of entering students.

By using some of the many communication tools and pathways available in our Information Age, and by taking full advantage of the desire to enhance student success, educators can make large strides towards ensuring that secondary students, and their teachers, are clear on what post-secondary instructors expect of them in first-year mathematics/statistics courses.

RECOMMENDATIONS

General Recommendations

1. That the preparation of students for post-secondary mathematics/statistics courses include significant opportunity for the development of the general mathematical proficiencies cited in this report (see page 12). Students have developed many isolated competencies in mathematics, but often lack experience in selecting those appropriate to the solution of a problem or in synthesising them logically. As much as possible, opportunities should be provided for students to learn logical approaches, modelling strategies and problem solving techniques prior to entering post-secondary mathematics/statistics.

2. That the instructional modes and the assessment of students preparing for post-secondary mathematics/statistics courses promote comfort with such skills as solving multi-step problems and as writing clear, well-organised solutions. Confidence in the ability to tackle longer problems or to write well-constructed solutions is enhanced through the successful completion of such projects. By expecting high standards of performance on problem-solving and solution writing assignments, teachers greatly assist students in nurturing confidence and competence in this regard.

3. That post-secondary mathematics/statistics departments be encouraged, independently and jointly, to communicate their mathematics/statistics proficiency expectations as a practicable guide for prospective students. Funding for the development of a provincial information brochure, and corresponding web site, listing in one place the entry-level proficiencies identified for post-secondary mathematics courses, is strongly recommended.

4. That the secondary and post-secondary mathematics instructors, through their respective provincial committees, work to ensure that the provincial final examinations for mathematics, and other assessment tools, evaluate the proficiencies/competencies seen as important for success in post-secondary courses. Also, the Ministry of Education should be encouraged to pursue this objective of achieving a better match of exit standards with post-secondary entry requirements.

5. That appropriate means be found to ensure that the findings in this Report are shared and discussed by secondary and post-secondary instructors of mathematics with the objective of promoting ongoing, constructive dialogue to address common issues. Student transition from secondary to post-secondary can only become seamless if all factors relating to this transition are frequently reviewed and are openly discussed. The onus and the challenge to find satisfactory means for engaging in such constructive dialogue rests with all sectors of the education system.

Calculus Recommendations

6. That students preparing for post-secondary calculus courses have a wide range of strong, mathematical proficiencies and that they have significant skill in choosing/combining various mathematical approaches or models to solve multi-stage problems. Higher level thinking skills, model building and solving multi-step problems were expectations emphasised particularly by respondents to the *Calculus* questionnaire. Those students anticipating taking

a post-secondary calculus course should be made aware of these expectations.

7. That students preparing for post-secondary calculus courses possess high levels of mathematical proficiency in the function concept, specific functions and graphing techniques. The manipulation of various functions in a calculus context requires strong conceptual and visualisation proficiencies. The development of these proficiencies by students is optimised through experience with many different functions and by seeing them in symbolic as well as graphical ways.

Introductory Statistics Recommendations

8. That teachers preparing students for post-secondary introductory statistics courses be strongly encouraged to provide a learning environment that will enable students to develop a positive attitude towards mathematics and an appreciation of the utility of quantitative approaches to everyday life situations. *Introductory Statistics* courses are frequently taken in order to fulfil requirements for some statistical literacy in career programs. Particularly for these students, it is important to develop an appreciation of the vital rôle mathematics plays in the many aspects of our lives.

9. That students preparing for post-secondary introductory statistics courses have developed some proficiency with descriptive statistics and with the elementary statistical functions of calculating or computing devices. Some students entering post-secondary statistics courses have encountered data analysis topics previously, while others express very little (or no) familiarity with this subject (see page 34). It is recommended that students be introduced to descriptive statistics and that, if possible, a related item be included on provincial final examinations. Also, students should be introduced to some calculating/ computing tools in the context of descriptive statistics.

10. That students preparing for post-secondary introductory statistics courses possess mathematical proficiencies in the areas of set theory, combinatorics (permutations, combinations and distinguishing them) and solving word problems. These proficiencies are very useful in *Introductory Statistics* and prior experience with them frees the student to concentrate on the applications in the course and to derive greater meaning from them.

11. That post-secondary mathematics/statistics departments seek means to communicate with one another, with other departments offering introductory statistics courses and with secondary school teachers concerning their individual proficiency expectations of introductory statistics students. The expectations of students in *Introductory Statistics* vary considerably among the post-secondary institutions. Although programs and departments enjoy full autonomy in the development and delivery of courses for their students, some awareness of practices elsewhere can be beneficial, particularly when course transfer credit becomes a factor. Thus, the establishment of a forum for communicating about issues connected with this course is recommended.

Mathematics for Elementary Education Recommendations

12. That students, teachers and parents have access to information that clearly describes the demands for first-year mathematics courses designed to meet Elementary Education

requirements, or designed as terminal mathematics courses to meet other program requirements. A positive attitude towards mathematics and an appreciation of the fact that mathematics is a significant part of almost every aspect of daily life are critical factors for success in such courses.

13. That students be made aware of the need to develop confidence and skill in a larger set of basic mathematical proficiencies than is presently the case in order to be successful in MFEE courses. This set of skills should include proficiency in: solving linear equations/inequalities, graphing and writing the equation of a line, working with polynomial, rational and exponential expressions, and having a basic understanding of plane geometry (see page 21). Respondents to the Survey commented on the overall weakness of some students in MFEE. They felt that if the students had even a small set of mathematical competencies (see page 38), then they could easily succeed in the course.

14. That students be encouraged to develop mathematical proficiencies in critical thinking and logic skills, and to maintain a positive attitude towards and a comfort with mathematics (see page 13). Conveying a positive, 'can do' attitude to young students is a critical aspect of teaching mathematics in the elementary school. For this reason, one of the major emphases of MFEE is the nurturing of such an attitude in MFEE students. This becomes an exceedingly difficult task when working with students strongly biased against mathematics. If students planning to become elementary school teachers or those involved in counselling such students can be made aware of these expectations through the wide distribution of some of the results of this Report, then new registrants in MFEE might well exhibit some improved general mathematical proficiencies.

15. That post-secondary departments involved in the delivery of MFEE courses commence a dialogue with appropriate stakeholders concerning their mathematical proficiency expectations of prospective MFEE students. Students can receive credit for MFEE by successfully completing a variety of other, acceptable courses, having their own entry criteria. Although this situation is not in itself seen as unwelcome, a complete review and a re-validation of current proficiency expectations for MFEE by stakeholders would reassure those involved in these courses (see page 38) that their local practices are generally acceptable.

APPENDIX A

Mathematical Proficiencies Ranked By Grade Level

In the following **Grade level groupings**, the mathematics sub-proficiencies of the Survey are ranked according to importance rating for entry to post-secondary courses. The Grade 11 grouping contains a few topics that are considered pre-Grade 11.

Grade 11 Proficiencies Ranked for CALCULUS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
L1	Understand and use function notation.		**4.00**
M1	Graph and write the equations for linear functions.		**4.00**
M2	Understand and use the slope of a line.		**4.00**
A1	Simplify polynomial expressions.		**3.96**
A2	Add and subtract polynomial expressions.		**3.96**
A3	Multiply and simplify polynomial expressions.		**3.96**
C1	Use the Laws of Exponents to simplify expressions.		**3.96**
E1	Solve linear equations		**3.96**
C2	Write radical expressions in exponential form.		**3.92**
F3	Understand and use the Pythagorean Theorem.		**3.92**
C3	Apply the Laws of Exponents to expressions with Rational exponents.		**3.88**
A4	Factor binomials and trinomials.		**3.83**
B2	Multiply and divide simple rational expressions.		**3.83**
B3	Add and subtract rational expressions.		**3.83**
H1	Understand and apply the primary trig ratios as related to triangles.		**3.83**
L3	Graph basic **R** $\rightarrow$ **R** functions like linear, absolute value, etc.	On Cartesian plane.	**3.83**
E5	Solve quadratic equations using the quadratic formula.		**3.81**
B1	Simplify rational expressions with polynomial numerators and denominators.		**3.79**
S1	Understand and use the primary trig ratios as related to angles of rotation.		**3.79**
N1	Graph and write the equations for given quadratic functions.		**3.75**
L2	Identify relations that are not functions.		**3.73**
D1	Simplify radical expressions with index 2.	Square roots.	**3.71**
D2	Apply the four basic operations to radical expressions with index 2.	Add, subtract, multiply, divide.	**3.71**
E8	Solve systems of linear equations in two variables.		**3.71**
E11	Solve equations containing rational expressions.		**3.67**
B4	Simplify complex fractions.		**3.54**
S2	Identify reference angles.		**3.54**
E7	Solve equations containing radicals with index 2.		**3.50**
M3	Write direct variations as equations.		**3.48**
H2	Know the trig ratios for special angles as numerical values.	0^o,30^o,45^o,60^o,90^o	**3.46**
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		**3.42**
A6	Divide polynomials with binomials of degree 1, using “long division”.		**3.31**
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function’s expression.	w.r.t. = with respect to	**3.29**
E4	Solve quadratic equations by completing the square.		**3.25**
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		**3.25**
A5	Factor polynomial expressions of more than 3 terms.		**3.15**
H4	Use the Laws of Sines and Cosines.		**3.10**
N2	Determine the max or min value of a quadratic function.	Without calculus.	**2.98**
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		**2.92**

Grade 11 Proficiencies Ranked for CALCULUS (continued)

E9	Graph solutions for systems of linear inequalities in 2 variables.		**2.67**
G3	Apply some basic properties of tangents to a circle.		**2.56**
H3	Use formulae involving trig ratios for the area of a triangle.		**2.40**
G1	Apply the basic properties of a chord of a circle.		**2.13**
G2	Apply the basic properties for angles of figures inscribed in a circle.	Inscribed triangles.	**2.08**
U4	Understand and apply the concept of set, subset and complement of a set.		**2.00**
J1	Organise and graph data from an experiment.		**0.88**
J2	Obtain, organise and graph sample data.		**0.83**
J3	Apply sample information to population information.		**0.54**
J4	Determine 90% confidence intervals.		**0.42**
J5	Select appropriate sampling methods.		**0.38**

Grade 12 Proficiencies Ranked for CALCULUS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
R1	Write exponential expressions in logarithmic form.		**3.79**
R2	Determine values of exponential and logarithmic functions.		**3.79**
S3	Understand and use the radian measure for angles.		**3.79**
S5	Graph and analyse the functions sine, cosine and tangent.		**3.79**
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		**3.75**
E15	Solve logarithmic and exponential equations.		**3.67**
L5	Determine the inverse of a function.	If it exists.	**3.67**
R4	Understand and apply the properties of logarithms.		**3.67**
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	**3.63**
E17	Solve trigonometric equations.		**3.54**
S8	Use the basic trig identities to re-write trig expressions.		**3.46**
R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	**3.42**
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		**3.38**
S9	Understand and use sum, difference and double angle identities.		**3.38**
S11	Use trigonometric identities to simplify trig expressions.		**3.38**
E3	Solve equations containing absolute value expressions.		**3.31**
P1	Work with the graph and the equation (+ features) of a circle.		**3.31**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	**3.31**
E6	Solve quadratic inequalities.		**3.27**
R6	Perform calculations found in practical settings using the techniques of logarithms.	Applications: sound, forgetting, etc.	**3.23**
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		**3.21**
P2	Work with the graph and the equation (+ features) of parabolas.		**3.17**
S4	Understand and use the 3 reciprocal trig ratios.	Cot(θ), Sec(θ), Csc(θ)	**3.17**
S10	Prove simple identities.		**3.00**
K3	Understand and use sigma notation.		**2.92**
E13	Solve polynomial equations of degree greater than 2.		**2.90**
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		**2.85**
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		**2.83**
K1	Understand and use arithmetic sequences and series formulae.		**2.75**
K2	Understand and use geometric sequences and series formulae.		**2.71**
F4	Prove simple geometric results.		**2.65**
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	**2.63**
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		**2.58**
Q4	Analyse the nature of the roots for some polynomial functions.		**2.52**

Grade 12 Proficiencies Ranked for CALCULUS (continued)

U3	Be familiar with writing simple proofs.		**2.50**
Q2	Apply the Factor and Remainder Theorems.		**2.48**
E14	Solve polynomial inequalities of degree greater than 2.		**2.31**
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	**2.08**
Q3	Understand and use notation for complex numbers.		**1.79**
A7	As in **A6**, using synthetic division.		**1.40**

Some of the Survey items in the table below are of generic in nature and cross Grade lines. Others are among those not included in the secondary school mathematics curriculum, but given a proficiency importance rating by respondents. All will be of interest to curriculum planners as will those proficiency items especially written in by respondents to the Survey questionnaire (see page 41).

Non-Grade Specific Proficiencies for CALCULUS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
E2	Solve linear inequalities		**3.85**
U7	Use a scientific calculator.	Log, Trig, etc.	**3.58**
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		**3.54**
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		**3.21**
D4	Apply the four basic operations to radical expressions with index *n* as in **D3**.		**3.17**
E12	Solve inequalities containing rational expressions.		**2.79**
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		**2.58**
E10	Solve systems of linear equations in 3 variables.		**2.56**
U8	Use a graphing calculator.		**2.21**
U5	Use the set operations: union, intersection.		**2.06**
H5	Understand and apply the concept of a vector.		**1.98**
H6	Determine the resultant of two vectors.		**1.94**
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		**1.85**
U6	Graph set interrelations using a Venn Diagram.		**1.52**
U9	Use a computer algebra system.	Maple, Mathematica, etc.	**1.42**
T1	Use limit notation and determine the limit value of given functions at a point.		**1.10**
T2	Determine whether a given sequence has a limit.		**1.10**
T3	Determine the derivative of a given polynomial function.		**0.94**
T4	Understand and use the power, product and quotient rules.		**0.94**
U10	Use a statistical software package.	SPSS, Minitab, etc.	**0.17**

Grade 11 Proficiencies Ranked for INTRODUCTORY STATISTICS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
M2	Understand and use the slope of a line.		**3.64**
M1	Graph and write the equations for linear functions.		**3.59**
U4	Understand and apply the concept of set, subset and complement of a set.		**2.89**
E1	Solve linear equations		**2.57**
J1	Organise and graph data from an experiment.		**2.43**
J2	Obtain, organise and graph sample data.		**2.34**
J3	Apply sample information to population information.		**2.20**

Grade 11 Proficiencies Ranked for INTRODUCTORY STATISTICS (continued)

M3	Write direct variations as equations.		**2.16**
J5	Select appropriate sampling methods.		**2.07**
J4	Determine 90% confidence intervals.		**2.02**
A1	Simplify polynomial expressions.		**1.95**
L1	Understand and use function notation.		**1.93**
A2	Add and subtract polynomial expressions.		**1.89**
A3	Multiply and simplify polynomial expressions.		**1.77**
D1	Simplify radical expressions with index 2.	Square roots.	**1.77**
C1	Use the Laws of Exponents to simplify expressions.		**1.66**
L3	Graph basic $\mathbf{R} \rightarrow \mathbf{R}$ functions like linear, absolute value, etc.	On Cartesian plane.	**1.61**
D2	Apply the four basic operations to radical expressions with index 2.	Add, subtract, multiply, divide.	**1.59**
C2	Write radical expressions in exponential form.		**1.36**
C3	Apply the Laws of Exponents to expressions with Rational exponents.		**1.30**
E8	Solve systems of linear equations in two variables.		**1.30**
B2	Multiply and divide simple rational expressions.		**1.23**
B3	Add and subtract rational expressions.		**1.23**
N1	Graph and write the equations for given quadratic functions.		**1.20**
E5	Solve quadratic equations using the quadratic formula.		**1.16**
B1	Simplify rational expressions with polynomial numerators and denominators.		**1.07**
A4	Factor binomials and trinomials.		**1.00**
L2	Identify relations that are not functions.		**0.98**
N2	Determine the max or min value of a quadratic function.	Without calculus	**0.93**
F3	Understand and use the Pythagorean Theorem.		**0.91**
E4	Solve quadratic equations by completing the square.		**0.89**
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function's expression.	w.r.t. = with respect to	**0.86**
E11	Solve equations containing rational expressions.		**0.84**
E7	Solve equations containing radicals with index 2.		**0.82**
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		**0.80**
E9	Graph solutions for systems of linear inequalities in 2 variables.		**0.80**
B4	Simplify complex fractions.		**0.70**
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		**0.64**
A5	Factor polynomial expressions of more than 3 terms.		**0.52**
A6	Divide polynomials with binomials of degree 1, using "long division".		**0.34**
H1	Understand and apply the primary trig ratios as related to triangles.		**0.32**
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		**0.27**
H2	Know the trig ratios for special angles as numerical values.	0º,30º,45º,60º,90º	**0.27**
G1	Apply the basic properties of a chord of a circle.		**0.23**
G2	Apply the basic properties for angles of figures inscribed in a circle.	Inscribed triangles.	**0.18**
G3	Apply some basic properties of tangents to a circle.		**0.18**
H3	Use formulae involving trig ratios for the area of a triangle.		**0.18**
S1	Understand and use the primary trig ratios as related to angles of rotation.		**0.18**
S2	Identify reference angles.		**0.18**
H4	Use the Laws of Sines and Cosines.		**0.14**

Grade 12 Proficiencies Ranked for INTRODUCTORY STATISTICS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
K3	Understand and use sigma notation.		**1.95**
E3	Solve equations containing absolute value expressions.		**1.52**
R1	Write exponential expressions in logarithmic form.		**1.20**

Grade 12 Proficiencies Ranked for INTRODUCTORY STATISTICS (continued)

R2	Determine values of exponential and logarithmic functions.		**1.16**
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		**1.14**
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	**1.11**
U3	Be familiar with writing simple proofs.		**1.02**
K1	Understand and use arithmetic sequences and series formulae.		**0.98**
K2	Understand and use geometric sequences and series formulae.		**0.98**
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	**0.93**
R4	Understand and apply the properties of logarithms.		**0.93**
R6	Perform calculations found in practical settings using the techniques of logarithms.	Applications: sound, forgetting, etc.	**0.89**
E15	Solve logarithmic and exponential equations.		**0.77**
L5	Determine the inverse of a function.	If it exists.	**0.70**
E6	Solve quadratic inequalities.		**0.66**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	**0.61**
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		**0.59**
R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	**0.52**
P1	Work with the graph and the equation (+ features) of a circle.		**0.50**
P2	Work with the graph and the equation (+ features) of parabolas.		**0.50**
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		**0.41**
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		**0.32**
E13	Solve polynomial equations of degree greater than 2.		**0.25**
A7	As in **A6**, using synthetic division.		**0.20**
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	**0.18**
Q2	Apply the Factor and Remainder Theorems.		**0.18**
Q4	Analyse the nature of the roots for some polynomial functions.		**0.18**
S5	Graph and analyse the functions sine, cosine and tangent.		**0.18**
E14	Solve polynomial inequalities of degree greater than 2.		**0.14**
F4	Prove simple geometric results.		**0.14**
Q3	Understand and use notation for complex numbers.		**0.14**
S3	Understand and use the radian measure for angles.		**0.14**
E17	Solve trigonometric equations.		**0.09**
S4	Understand and use the 3 reciprocal trig ratios.	$\text{Cot}(\theta)$, $\text{Sec}(\theta)$, $\text{Csc}(\theta)$	**0.09**
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		**0.09**
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		**0.09**
S8	Use the basic trig identities to re-write trig expressions.		**0.09**
S9	Understand and use sum, difference and double angle identities.		**0.09**
S10	Prove simple identities.		**0.09**
S11	Use trigonometric identities to simplify trig expressions.		**0.09**

Non-Grade Specific Proficiencies for INTRODUCTORY STATISTICS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		**3.07**
U5	Use the set operations: union, intersection.		**2.93**
U6	Graph set interrelations using a Venn Diagram.		**2.84**
U7	Use a scientific calculator.	Log, Trig, etc.	**2.73**
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		**2.25**

Non-Grade Specific Proficiencies for INTRODUCTORY STATISTICS (continued)

E2	Solve linear inequalities		**1.95**
U10	Use a statistical software package.	SPSS, Minitab, etc.	**1.82**
U8	Use a graphing calculator.		**0.93**
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		**0.84**
T1	Use limit notation and determine the limit value of given functions at a point.		**0.77**
D4	Apply the four basic operations to radical expressions with index *n* as in **D3**.		**0.61**
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		**0.57**
E10	Solve systems of linear equations in 3 variables.		**0.57**
T4	Understand and use the power, product and quotient rules.		**0.57**
T3	Determine the derivative of a given polynomial function.		**0.52**
U9	Use a computer algebra system.	Maple, Mathematica, etc.	**0.52**
E12	Solve inequalities containing rational expressions.		**0.50**
T2	Determine whether a given sequence has a limit.		**0.50**
H5	Understand and apply the concept of a vector.		**0.45**
H6	Determine the resultant of two vectors.		**0.32**

The 5 sub-proficiencies given an importance rating greater than **2** in the Additional Proficiencies table above are of particular interest for *Introductory Statistics*, given its overall rating profile in the Survey.

<u>Grade 11</u> Proficiencies Ranked for MATHEMATICS FOR ELEMENTARY EDUCATION

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
E1	Solve linear equations		**3.31**
A2	Add and subtract polynomial expressions.		**2.50**
F3	Understand and use the Pythagorean Theorem.		**2.50**
A1	Simplify polynomial expressions.		**2.38**
C1	Use the Laws of Exponents to simplify expressions.		**2.38**
D1	Simplify radical expressions with index 2.	Square roots.	**2.38**
A3	Multiply and simplify polynomial expressions.		**2.31**
M1	Graph and write the equations for linear functions.		**2.31**
M2	Understand and use the slope of a line.		**2.31**
U4	Understand and apply the concept of set, subset and complement of a set.		**2.31**
E8	Solve systems of linear equations in two variables.		**2.19**
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		**2.06**
B2	Multiply and divide simple rational expressions.		**2.00**
B3	Add and subtract rational expressions.		**1.94**
D2	Apply the four basic operations to radical expressions with index 2.	Add, subtract, multiply, divide.	**1.94**
L1	Understand and use function notation.		**1.88**
A4	Factor binomials and trinomials.		**1.81**
B1	Simplify rational expressions with polynomial numerators and denominators.		**1.75**
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		**1.69**
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		**1.56**
B4	Simplify complex fractions.		**1.50**
C2	Write radical expressions in exponential form.		**1.50**
C3	Apply the Laws of Exponents to expressions with Rational exponents.		**1.50**
J1	Organise and graph data from an experiment.		**1.50**
L2	Identify relations that are not functions.		**1.50**

Grade 11 Proficiencies Ranked for MATH FOR ELEMENTARY EDUCATION (continued)

L3	Graph basic **R** $\longrightarrow$ **R** functions like linear, absolute value, etc.	On Cartesian plane.	**1.50**
M3	Write direct variations as equations.		**1.44**
E9	Graph solutions for systems of linear inequalities in 2 variables.		**1.31**
E11	Solve equations containing rational expressions.		**1.25**
E5	Solve quadratic equations using the quadratic formula.		**1.19**
J2	Obtain, organise and graph sample data.		**1.19**
E4	Solve quadratic equations by completing the square.		**1.13**
N2	Determine the max or min value of a quadratic function.	Without calculus	**1.00**
A5	Factor polynomial expressions of more than 3 terms.		**0.94**
A6	Divide polynomials with binomials of degree 1, using "long division".		**0.94**
G1	Apply the basic properties of a chord of a circle.		**0.94**
N1	Graph and write the equations for given quadratic functions.		**0.94**
G3	Apply some basic properties of tangents to a circle.		**0.88**
G2	Apply the basic properties for angles of figures inscribed in a circle.	Inscribed triangles.	**0.81**
J3	Apply sample information to population information.		**0.81**
E7	Solve equations containing radicals with index 2.		**0.75**
H1	Understand and apply the primary trig ratios as related to triangles.		**0.75**
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function's expression.	w.r.t. = with respect to	**0.63**
H2	Know the trig ratios for special angles as numerical values.	0º,30º,45º,60º,90º	**0.56**
J5	Select appropriate sampling methods.		**0.56**
H3	Use formulae involving trig ratios for the area of a triangle.		**0.38**
J4	Determine 90% confidence intervals.		**0.38**
S1	Understand and use the primary trig ratios as related to angles of rotation.		**0.19**
H4	Use the Laws of Sines and Cosines.		**0.13**
S2	Identify reference angles.		**0.13**

Grade 12 Proficiencies Ranked for MATHEMATICS FOR ELEMENTARY EDUCATION

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
F4	Prove simple geometric results.		**1.56**
E3	Solve equations containing absolute value expressions.		**1.13**
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		**1.06**
K1	Understand and use arithmetic sequences and series formulae.		**1.00**
U3	Be familiar with writing simple proofs.		**1.00**
K2	Understand and use geometric sequences and series formulae.		**0.81**
R1	Write exponential expressions in logarithmic form.		**0.63**
A7	As in **A6**, using synthetic division.		**0.56**
E6	Solve quadratic inequalities.		**0.56**
E15	Solve logarithmic and exponential equations.		**0.56**
R2	Determine values of exponential and logarithmic functions.		**0.56**
R6	Perform calculations found in practical settings using the techniques of logarithms.	Applications: sound, forgetting, etc.	**0.56**
E13	Solve polynomial equations of degree greater than 2.		**0.50**
K3	Understand and use sigma notation.		**0.50**
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	**0.50**
P1	Work with the graph and the equation (+ features) of a circle.		**0.44**
R4	Understand and apply the properties of logarithms.		**0.44**
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	**0.44**
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	**0.38**
P2	Work with the graph and the equation (+ features) of parabolas.		**0.38**
R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	**0.38**
E14	Solve polynomial inequalities of degree greater than 2.		**0.25**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	**0.25**

Grade 12 Proficiencies Ranked for MATH FOR ELEMENTARY EDUCATION (continued)

S3	Understand and use the radian measure for angles.		**0.25**
E17	Solve trigonometric equations.		**0.19**
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		**0.19**
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		**0.19**
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		**0.13**
Q3	Understand and use notation for complex numbers.		**0.13**
S4	Understand and use the 3 reciprocal trig ratios.	Cot(θ), Sec(θ), Csc(θ)	**0.13**
S8	Use the basic trig identities to re-write trig expressions.		**0.07**
L5	Determine the inverse of a function.	If it exists.	**0.06**
Q2	Apply the Factor and Remainder Theorems.		**0.06**
S5	Graph and analyse the functions sine, cosine and tangent.		**0.06**
Q4	Analyse the nature of the roots for some polynomial functions.		**0.00**
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		**0.00**
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		**0.00**
S9	Understand and use sum, difference and double angle identities.		**0.00**
S10	Prove simple identities.		**0.00**
S11	Use trigonometric identities to simplify trig expressions.		**0.00**

Non-Grade Specific Proficiencies for MATHEMATICS FOR ELEMENTARY EDUCATION

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
E2	Solve linear inequalities		**2.63**
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		**2.63**
U5	Use the set operations: union, intersection.		**2.44**
U6	Graph set interrelations using a Venn Diagram.		**2.38**
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		**1.31**
U7	Use a scientific calculator.	Log, Trig, etc.	**1.31**
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		**1.13**
D4	Apply the four basic operations to radical expressions with index *n* as in **D3**.		**0.94**
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		**0.94**
E10	Solve systems of linear equations in 3 variables.		**0.88**
E12	Solve inequalities containing rational expressions.		**0.56**
U8	Use a graphing calculator.		**0.31**
U10	Use a statistical software package.	SPSS, Minitab, etc.	**0.19**
H5	Understand and apply the concept of a vector.		**0.13**
U9	Use a computer algebra system.	Maple, Mathematica	**0.13**
H6	Determine the resultant of two vectors.		**0.06**
T1	Use limit notation and determine the limit value of given functions at a point.		**0.00**
T2	Determine whether a given sequence has a limit.		**0.00**
T3	Determine the derivative of a given polynomial function.		**0.00**
T4	Understand and use the power, product and quotient rules.		**0.00**

APPENDIX B

Sub-Proficiencies Ranked

Sub-Proficiencies of Survey Ranked for CALCULUS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
L1	Understand and use function notation.		**4.00**
M1	Graph and write the equations for linear functions.		**4.00**
M2	Understand and use the slope of a line.		**4.00**
A1	Simplify polynomial expressions.		**3.96**
A2	Add and subtract polynomial expressions.		**3.96**
A3	Multiply and simplify polynomial expressions.		**3.96**
C1	Use the Laws of Exponents to simplify expressions.		**3.96**
E1	Solve linear equations.		**3.96**
C2	Write radical expressions in exponential form.		**3.92**
F3	Understand and use the Pythagorean Theorem.		**3.92**
C3	Apply the Laws of Exponents to expressions with rational exponents.		**3.88**
E2	Solve linear inequalities		**3.85**
A4	Factor binomials and trinomials.		**3.83**
B2	Multiply and divide simple rational expressions.		**3.83**
B3	Add and subtract rational expressions.		**3.83**
H1	Understand and apply the primary trig ratios as related to triangles.		**3.83**
L3	Graph basic $\mathbf{R} \rightarrow \mathbf{R}$ functions like linear, absolute value, etc.	On Cartesian plane.	**3.83**
E5	Solve quadratic equations using the quadratic formula.		**3.81**
B1	Simplify rational expressions with polynomial numerators and denominators.		**3.79**
R1	Write exponential expressions in logarithmic form.		**3.79**
R2	Determine values of exponential and logarithmic functions.		**3.79**
S1	Understand and use the primary trig ratios as related to angles of rotation.		**3.79**
S3	Understand and use the radian measure for angles.		**3.79**
S5	Graph and analyse the functions sine, cosine and tangent.		**3.79**
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		**3.75**
N1	Graph and write the equations for given quadratic functions.		**3.75**
L2	Identify relations that are not functions.		**3.73**
D1	Simplify radical expressions with index 2.	Square roots.	**3.71**
D2	Apply the four basic operations to radical expressions with index 2.	Add, subtract, multiply, divide.	**3.71**
E8	Solve systems of linear equations in two variables.		**3.71**
E11	Solve equations containing rational expressions.		**3.67**
E15	Solve logarithmic and exponential equations.		**3.67**
L5	Determine the inverse of a function.	If it exists.	**3.67**
R4	Understand and apply the properties of logarithms.		**3.67**
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	**3.63**
U7	Use a scientific calculator.	Log, Trig, etc.	**3.58**
B4	Simplify complex fractions.		**3.54**
E17	Solve trigonometric equations.		**3.54**
S2	Identify reference angles.		**3.54**
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		**3.54**
E7	Solve equations containing radicals with index 2.		**3.50**
M3	Write direct variations as equations.		**3.48**
H2	Know the trig ratios for special angles as numerical values.	0º,30º,45º,60º,90º	**3.46**
S8	Use the basic trig identities to re-write trig expressions.		**3.46**
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		**3.42**

Sub-Proficiencies of Survey Ranked for CALCULUS (Continued)

R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	**3.42**
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		**3.38**
S9	Understand and use sum, difference and double angle identities.		**3.38**
S11	Use trigonometric identities to simplify trig expressions.		**3.38**
A6	Divide polynomials with binomials of degree 1, using "long division".		**3.31**
E3	Solve equations containing absolute value expressions.		**3.31**
P1	Work with the graph and the equation (+ features) of a circle.		**3.31**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	**3.31**
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function's expression.	w.r.t. = with respect to	**3.29**
E6	Solve quadratic inequalities.		**3.27**
E4	Solve quadratic equations by completing the square.		**3.25**
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		**3.25**
R6	Perform calculations found in practical settings using the techniques of logarithms.	Applications: sound, forgetting, etc.	**3.23**
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		**3.21**
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		**3.21**
D4	Apply the four basic operations to radical expressions with index *n* as in **D3**.		**3.17**
P2	Work with the graph and the equation (+ features) of parabolas.		**3.17**
S4	Understand and use the 3 reciprocal trig ratios.	Cot(θ), Sec(θ), Csc(θ)	**3.17**
A5	Factor polynomial expressions of more than 3 terms.		**3.15**
H4	Use the Laws of Sines and Cosines.		**3.10**
S10	Prove simple identities.		**3.00**
N2	Determine the max or min value of a quadratic function.	Without calculus	**2.98**
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		**2.92**
K3	Understand and use sigma notation.		**2.92**
E13	Solve polynomial equations of degree greater than 2.		**2.90**
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		**2.85**
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		**2.83**
E12	Solve inequalities containing rational expressions.		**2.79**
K1	Understand and use arithmetic sequences and series formulae.		**2.75**
K2	Understand and use geometric sequences and series formulae.		**2.71**
E9	Graph solutions for systems of linear inequalities in 2 variables.		**2.67**
F4	Prove simple geometric results.		**2.65**
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	**2.63**
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		**2.58**
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		**2.58**
E10	Solve systems of linear equations in 3 variables.		**2.56**
G3	Apply some basic properties of tangents to a circle.		**2.56**
Q4	Analyse the nature of the roots for some polynomial functions.		**2.52**
U3	Be familiar with writing simple proofs.		**2.50**
Q2	Apply the Factor and Remainder Theorems.		**2.48**
H3	Use formulae involving trig ratios for the area of a triangle.		**2.40**
E14	Solve polynomial inequalities of degree greater than 2.		**2.31**
U8	Use a graphing calculator.		**2.21**
G1	Apply the basic properties of a chord of a circle.		**2.13**
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	**2.08**
G2	Apply the basic properties for angles of figures inscribed in a circle.	Inscribed triangles.	**2.08**
U5	Use the set operations: union, intersection.		**2.06**
U4	Understand and apply the concept of set, subset and complement of a set.		**2.00**
H5	Understand and apply the concept of a vector.		**1.98**
H6	Determine the resultant of two vectors.		**1.94**

Sub-Proficiencies of Survey Ranked for CALCULUS (Continued)

U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		**1.85**
Q3	Understand and use notation for complex numbers.		**1.79**
U6	Graph set interrelations using a Venn Diagram.		**1.52**
U9	Use a computer algebra system.	Maple, Mathematica.	**1.42**
A7	As in **A6**, using synthetic division.		**1.40**
T1	Use limit notation and determine the limit value of given functions at a point.		**1.10**
T2	Determine whether a given sequence has a limit.		**1.10**
T3	Determine the derivative of a given polynomial function.		**0.94**
T4	Understand and use the power, product and quotient rules.		**0.94**
J1	Organise and graph data from an experiment.		**0.88**
J2	Obtain, organise and graph sample data.		**0.83**
J3	Apply sample information to population information.		**0.54**
J4	Determine 90% confidence intervals.		**0.42**
J5	Select appropriate sampling methods.		**0.38**
U10	Use a statistical software package.	SPSS, Minitab, etc.	**0.17**

Sub-Proficiencies of Survey Ranked for INTRODUCTORY STATISTICS

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
M2	Understand and use the slope of a line.		**3.64**
M1	Graph and write the equations for linear functions.		**3.59**
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		**3.07**
U5	Use the set operations: union, intersection.		**2.93**
U4	Understand and apply the concept of set, subset and complement of a set.		**2.89**
U6	Graph set interrelations using a Venn Diagram.		**2.84**
U7	Use a scientific calculator.	Log, Trig, etc.	**2.73**
E1	Solve linear equations.		**2.57**
J1	Organise and graph data from an experiment.		**2.43**
J2	Obtain, organise and graph sample data.		**2.34**
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		**2.25**
J3	Apply sample information to population information.		**2.20**
M3	Write direct variations as equations.		**2.16**
J5	Select appropriate sampling methods.		**2.07**
J4	Determine 90% confidence intervals.		**2.02**
A1	Simplify polynomial expressions.		**1.95**
E2	Solve linear inequalities.		**1.95**
K3	Understand and use sigma notation.		**1.95**
L1	Understand and use function notation.		**1.93**
A2	Add and subtract polynomial expressions.		**1.89**
U11	Use a statistical software package.	SPSS, Minitab, etc.	**1.82**
A3	Multiply and simplify polynomial expressions.		**1.77**
D1	Simplify radical expressions with index 2.	Square roots.	**1.77**
C1	Use the Laws of Exponents to simplify expressions.		**1.66**
L3	Graph basic $\mathbf{R} \longrightarrow \mathbf{R}$ functions like linear, absolute value, etc.	On Cartesian plane.	**1.61**
D2	Apply the four basic operations to radical expressions with index 2.	Add, subtract, multiply, divide.	**1.59**
E3	Solve equations containing absolute value expressions.		**1.52**
C2	Write radical expressions in exponential form.		**1.36**
C3	Apply the Laws of Exponents to expressions with Rational exponents.		**1.30**
E8	Solve systems of linear equations in two variables.		**1.30**
B2	Multiply and divide simple rational expressions.		**1.23**
B3	Add and subtract rational expressions.		**1.23**
N1	Graph and write the equations for given quadratic functions.		**1.20**

Sub-Proficiencies of Survey Ranked for INTRODUCTORY STATISTICS (Continued)

R1	Write exponential expressions in logarithmic form.		**1.20**
E5	Solve quadratic equations using the quadratic formula.		**1.16**
R2	Determine values of exponential and logarithmic functions.		**1.16**
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		**1.14**
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	**1.11**
B1	Simplify rational expressions with polynomial numerators and denominators.		**1.07**
U3	Be familiar with writing simple proofs.		**1.02**
A4	Factor binomials and trinomials.		**1.00**
K1	Understand and use arithmetic sequences and series formulae.		**0.98**
K2	Understand and use geometric sequences and series formulae.		**0.98**
L2	Identify relations that are not functions.		**0.98**
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	**0.93**
N2	Determine the max or min value of a quadratic function.	Without calculus	**0.93**
R4	Understand and apply the properties of logarithms.		**0.93**
U8	Use a graphing calculator.		**0.93**
F3	Understand and use the Pythagorean Theorem.		**0.91**
E4	Solve quadratic equations by completing the square.		**0.89**
R6	Perform calculations found in practical settings using the techniques of logarithms.	Applications: sound, forgetting, etc.	**0.89**
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function's expression.	w.r.t. = with respect to	**0.86**
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		**0.84**
E11	Solve equations containing rational expressions.		**0.84**
E7	Solve equations containing radicals with index 2.		**0.82**
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		**0.80**
E9	Graph solutions for systems of linear inequalities in 2 variables.		**0.80**
E15	Solve logarithmic and exponential equations.		**0.77**
T1	Use limit notation and determine the limit value of given functions at a point.		**0.77**
B4	Simplify complex fractions.		**0.70**
L5	Determine the inverse of a function.	If it exists.	**0.70**
E6	Solve quadratic inequalities.		**0.66**
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		**0.64**
D4	Apply the four basic operations to radical expressions with index *n* as in **D3**.		**0.61**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	**0.61**
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		**0.59**
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		**0.57**
E10	Solve systems of linear equations in 3 variables.		**0.57**
T4	Understand and use the power, product and quotient rules.		**0.57**
A5	Factor polynomial expressions of more than 3 terms.		**0.52**
R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	**0.52**
T3	Determine the derivative of a given polynomial function.		**0.52**
U9	Use a computer algebra system.	Maple, Mathematica.	**0.52**
E12	Solve inequalities containing rational expressions.		**0.50**
P1	Work with the graph and the equation (+ features) of a circle.		**0.50**
P2	Work with the graph and the equation (+ features) of parabolas.		**0.50**
T2	Determine whether a given sequence has a limit.		**0.50**
H5	Understand and apply the concept of a vector.		**0.45**
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		**0.41**
A6	Divide polynomials with binomials of degree 1, using "long division".		**0.34**
H1	Understand and apply the primary trig ratios as related to triangles.		**0.32**
H6	Determine the resultant of two vectors.		**0.32**
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		**0.32**

Sub-Proficiencies of Survey Ranked for INTRODUCTORY STATISTICS (Continued)

F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		**0.27**
H2	Know the trig ratios for special angles as numerical values.	0º,30º,45º,60º,90º	**0.27**
E13	Solve polynomial equations of degree greater than 2.		**0.25**
G1	Apply the basic properties of a chord of a circle.		**0.23**
A7	As in **A6**, using synthetic division.		**0.20**
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	**0.18**
G2	Apply the basic properties for angles of figures inscribed in a circle.	Inscribed triangles.	**0.18**
G3	Apply some basic properties of tangents to a circle.		**0.18**
H3	Use formulae involving trig ratios for the area of a triangle.		**0.18**
Q2	Apply the Factor and Remainder Theorems.		**0.18**
Q4	Analyse the nature of the roots for some polynomial functions.		**0.18**
S1	Understand and use the primary trig ratios as related to angles of rotation.		**0.18**
S2	Identify reference angles.		**0.18**
S5	Graph and analyse the functions sine, cosine and tangent.		**0.18**
E14	Solve polynomial inequalities of degree greater than 2.		**0.14**
F4	Prove simple geometric results.		**0.14**
H4	Use the Laws of Sines and Cosines.		**0.14**
Q3	Understand and use notation for complex numbers.		**0.14**
S3	Understand and use the radian measure for angles.		**0.14**
E17	Solve trigonometric equations.		**0.09**
S4	Understand and use the 3 reciprocal trig ratios.	Cot(θ), Sec(θ), Csc(θ)	**0.09**
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		**0.09**
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		**0.09**
S8	Use the basic trig identities to re-write trig expressions.		**0.09**
S9	Understand and use sum, difference and double angle identities.		**0.09**
S10	Prove simple identities.		**0.09**
S11	Use trigonometric identities to simplify trig expressions.		**0.09**

Sub-Proficiencies of Survey Ranked for MATHEMATICS FOR ELEMENTARY EDUCATION

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
E1	Solve linear equations.		**3.31**
E2	Solve linear inequalities		**2.63**
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		**2.63**
A2	Add and subtract polynomial expressions.		**2.50**
F3	Understand and use the Pythagorean Theorem.		**2.50**
U5	Use the set operations: union, intersection.		**2.44**
A1	Simplify polynomial expressions.		**2.38**
C1	Use the Laws of Exponents to simplify expressions.		**2.38**
D1	Simplify radical expressions with index 2.	Square roots.	**2.38**
U6	Graph set interrelations using a Venn Diagram.		**2.38**
A3	Multiply and simplify polynomial expressions.		**2.31**
M1	Graph and write the equations for linear functions.		**2.31**
M2	Understand and use the slope of a line.		**2.31**
U4	Understand and apply the concept of set, subset and complement of a set.		**2.31**
E8	Solve systems of linear equations in two variables.		**2.19**

Sub-Proficiencies of Survey Ranked for MATH FOR ELEMENTARY EDUCATION (Continued)

F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		**2.06**
B2	Multiply and divide simple rational expressions.		**2.00**
B3	Add and subtract rational expressions.		**1.94**
D2	Apply the four basic operations to radical expressions with index 2.	Add, subtract, multiply, divide.	**1.94**
L1	Understand and use function notation.		**1.88**
A4	Factor binomials and trinomials.		**1.81**
B1	Simplify rational expressions with polynomial numerators and denominators.		**1.75**
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		**1.69**
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		**1.56**
F4	Prove simple geometric results.		**1.56**
B4	Simplify complex fractions.		**1.50**
C2	Write radical expressions in exponential form.		**1.50**
C3	Apply the Laws of Exponents to expressions with Rational exponents.		**1.50**
J1	Organise and graph data from an experiment.		**1.50**
L2	Identify relations that are not functions.		**1.50**
L3	Graph basic **R** $\longrightarrow$ **R** functions like linear, absolute value, etc.	On Cartesian plane.	**1.50**
M3	Write direct variations as equations.		**1.44**
E9	Graph solutions for systems of linear inequalities in 2 variables.		**1.31**
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		**1.31**
U7	Use a scientific calculator.	Log, Trig, etc.	**1.31**
E11	Solve equations containing rational expressions.		**1.25**
E5	Solve quadratic equations using the quadratic formula.		**1.19**
J2	Obtain, organise and graph sample data.		**1.19**
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		**1.13**
E3	Solve equations containing absolute value expressions.		**1.13**
E4	Solve quadratic equations by completing the square.		**1.13**
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		**1.06**
K1	Understand and use arithmetic sequences and series formulae.		**1.00**
N2	Determine the max or min value of a quadratic function.	Without calculus	**1.00**
U3	Be familiar with writing simple proofs.		**1.00**
A5	Factor polynomial expressions of more than 3 terms.		**0.94**
A6	Divide polynomials with binomials of degree 1, using “long division”.		**0.94**
D4	Apply the four basic operations to radical expressions with index *n* as in **D3**.		**0.94**
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		**0.94**
G1	Apply the basic properties of a chord of a circle.		**0.94**
N1	Graph and write the equations for given quadratic functions.		**0.94**
E10	Solve systems of linear equations in 3 variables.		**0.88**
G3	Apply some basic properties of tangents to a circle.		**0.88**
G2	Apply the basic properties for angles of figures inscribed in a circle.	Inscribed triangles.	**0.81**
J3	Apply sample information to population information.		**0.81**
K2	Understand and use geometric sequences and series formulae.		**0.81**
E7	Solve equations containing radicals with index 2.		**0.75**
H1	Understand and apply the primary trig ratios as related to triangles.		**0.75**
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function’s expression.	w.r.t. = with respect to	**0.63**
R1	Write exponential expressions in logarithmic form.		**0.63**
A7	As in A6, using synthetic division.		**0.56**
E6	Solve quadratic inequalities.		**0.56**
E12	Solve inequalities containing rational expressions.		**0.56**
E15	Solve logarithmic and exponential equations.		**0.56**
H2	Know the trig ratios for special angles as numerical values.	0^o,30^o,45^o,60^o,90^o	**0.56**
J5	Select appropriate sampling methods.		**0.56**
R2	Determine values of exponential and logarithmic functions.		**0.56**

Sub-Proficiencies of Survey Ranked for MATH FOR ELEMENTARY EDUCATION (Continued)

R6	Perform calculations found in practical settings using the techniques of logarithms.	Applications: sound, forgetting, etc.	**0.56**
E13	Solve polynomial equations of degree greater than 2.		**0.50**
K3	Understand and use sigma notation.		**0.50**
K4	Apply the formula for an infinite geometric series.	$S = a/(1 - r)$	**0.50**
P1	Work with the graph and the equation (+ features) of a circle.		**0.44**
R4	Understand and apply the properties of logarithms.		**0.44**
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	**0.44**
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	**0.38**
H3	Use formulae involving trig ratios for the area of a triangle.		**0.38**
J4	Determine 90% confidence intervals.		**0.38**
P2	Work with the graph and the equation (+ features) of parabolas.		**0.38**
R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	**0.38**
U8	Use a graphing calculator.		**0.31**
E14	Solve polynomial inequalities of degree greater than 2.		**0.25**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	**0.25**
S3	Understand and use the radian measure for angles.		**0.25**
E17	Solve trigonometric equations.		**0.19**
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		**0.19**
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		**0.19**
S1	Understand and use the primary trig ratios as related to angles of rotation.		**0.19**
U10	Use a statistical software package.	SPSS, Minitab, etc.	**0.19**
H4	Use the Laws of Sines and Cosines.		**0.13**
H5	Understand and apply the concept of a vector.		**0.13**
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		**0.13**
Q3	Understand and use notation for complex numbers.		**0.13**
S2	Identify reference angles.		**0.13**
S4	Understand and use the 3 reciprocal trig ratios.	$\text{Cot}(\theta)$, $\text{Sec}(\theta)$, $\text{Csc}(\theta)$	**0.13**
U9	Use a computer algebra system.	Maple, Mathematica.	**0.13**
S8	Use the basic trig identities to re-write trig expressions.		**0.07**
H6	Determine the resultant of two vectors.		**0.06**
L5	Determine the inverse of a function.	If it exists.	**0.06**
Q2	Apply the Factor and Remainder Theorems.		**0.06**
S5	Graph and analyse the functions sine, cosine and tangent.		**0.06**
Q4	Analyse the nature of the roots for some polynomial functions.		**0.00**
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		**0.00**
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		**0.00**
S9	Understand and use sum, difference and double angle identities.		**0.00**
S10	Prove simple identities.		**0.00**
S11	Use trigonometric identities to simplify trig expressions.		**0.00**
T1	Use limit notation and determine the limit value of given functions at a point.		**0.00**
T2	Determine whether a given sequence has a limit.		**0.00**
T3	Determine the derivative of a given polynomial function.		**0.00**
T4	Understand and use the power, product and quotient rules.		**0.00**

APPENDIX C

Survey Questionnaire Results – Part A

The Mathematics Proficiencies Survey contained three questionnaires—one each for *Calculus*, *Introductory Statistics* and *Mathematics for Elementary Education*. Each of the 24 post-secondary educational institutions in the Survey completed questionnaires only for those courses offered by the institution. All offer at least one *Calculus (Calc)* course, all but one offer an *Introductory Statistics (Intro Stats)* course and 16 institutions provide a *Mathematics for Elementary Education (MFEE)* or one transferring as such.

Respondents were asked to give a proficiency or competency **Importance Rating** to each topic and each Proficiency Category listed in Part A. They used an integral importance scale of 4 to 0 where **4** can be interpreted as **vitally important** and **0** as **not important**. For interpretative purposes, the Steering Committee suggested that any proficiency receiving an importance rating of **2**, **marginally important**, or greater should be regarded as a significant competency for any entering student.

Mathematical proficiencies were grouped into Categories with respondents first assessing the overall importance of the Category and then rating each of the sub-proficiencies within the Category. Further, the questionnaire encouraged respondents to suggest sub-proficiencies they felt were not sufficiently highlighted. These results are reported beginning on page 41.

Part A (columns 1 to 3) of the Survey questionnaire appears below with the mean of the importance ratings given by respondents for each of the courses of the Survey appearing in columns 4 to 5. The Proficiency Category is identified in the first line of each Table and the last line provides the **Category Mean**, the mean of the ratings for all sub-proficiencies within the Category.

Survey Questionnaire (Part A) and Mean of Responses

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
A	**Understand and use Polynomial Expressions.**		**3.96**	**1.95**	**2.25**
A1	Simplify polynomial expressions.		3.96	1.95	2.38
A2	Add and subtract polynomial expressions.		3.96	1.89	2.50
A3	Multiply and simplify polynomial expressions.		3.96	1.77	2.31
A4	Factor binomials and trinomials.		3.83	1.00	1.81
A5	Factor polynomial expressions of more than 3 terms.		3.15	0.52	0.94
A6	Divide polynomials with binomials of degree 1, using "long division".		3.31	0.34	0.94
A7	As in **A6**, using synthetic division.		1.40	0.20	0.56
		Category Mean	**3.37**	**1.10**	**1.63**

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
B	**Understand and use Rational Expressions.**		**3.83**	**1.43**	**2.00**
B1	Simplify rational expressions with polynomial numerators and denominators.		3.79	1.07	1.75
B2	Multiply and divide simple rational expressions.		3.83	1.23	2.00
B3	Add and subtract rational expressions.		3.83	1.23	1.94
B4	Simplify complex fractions.		3.54	0.70	1.50
		Category Mean	**3.75**	**1.06**	**1.80**
C	**Understand and use Exponential Expressions.**		**3.96**	**1.82**	**2.25**
C1	Use the Laws of Exponents to simplify expressions.		3.96	1.66	2.38
C2	Write radical expressions in exponential form.		3.92	1.36	1.50
C3	Apply the Laws of Exponents to expressions with Rational exponents.		3.88	1.30	1.50
C4	Apply Laws of Exponents to expressions with Real (including Irrational) exponents.		3.75	1.14	1.06
		Category Mean	**3.88**	**1.36**	**1.61**
D	**Understand and use Radical Expressions.**		**3.75**	**1.89**	**2.25**
D1	Simplify radical expressions with index 2.	Square roots.	3.71	1.77	2.38
D2	Apply the four basic operations to radical expressions with index 2.	Adding, subtracting, multiplying, dividing.	3.71	1.59	1.94
D3	Simplify radical expressions with index *n*, where *n* is an integer other than 2.		3.21	0.84	1.13
D4	Apply the four basic operations to radical expressions with index *n* as in **D3.**		3.17	0.61	0.94
D5	Rationalise the numerator or denominator of expressions containing radicals with index 2.		3.42	0.80	1.56
D6	Rationalise the numerator or denominator of expressions such as in **D5** with index *n*.		2.58	0.57	0.94
		Category Mean	**3.30**	**1.03**	**1.48**

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
E	**Solve Equations and Inequalities.**		**3.92**	**2.30**	**2.94**
E1	Solve linear equations		3.96	2.57	3.31
E2	Solve linear inequalities		3.85	1.95	2.63
E3	Solve equations containing absolute value expressions.		3.31	1.52	1.13
E4	Solve quadratic equations by completing the square.		3.25	0.89	1.13
E5	Solve quadratic equations using the quadratic formula.		3.81	1.16	1.19
E6	Solve quadratic inequalities.		3.27	0.66	0.56
E7	Solve equations containing radicals with index 2.		3.50	0.82	0.75
E8	Solve systems of linear equations in two variables.		3.71	1.30	2.19
E9	Graph solutions for systems of linear inequalities in 2 variables.		2.67	0.80	1.31
E10	Solve systems of linear equations in 3 variables.		2.56	0.57	0.88
E11	Solve equations containing rational expressions.		3.67	0.84	1.25
E12	Solve inequalities containing rational expressions.		2.79	0.50	0.56
E13	Solve polynomial equations of degree greater than 2.		2.90	0.25	0.50
E14	Solve polynomial inequalities of degree greater than 2.		2.31	0.14	0.25
E15	Solve logarithmic and exponential equations.		3.67	0.77	0.56
E16	Solve tailored systems of quadratic equations in 2 variables.	Few linear terms.	2.08	0.18	0.38
E17	Solve trigonometric equations.		3.54	0.09	0.19
		Category Mean	**3.23**	**0.88**	**1.10**

F	**Understand and use the Geometry of Lines and Points.**		**3.50**	**1.23**	**2.19**
F1	Understand and apply the angle relations resulting from two intersecting lines and from parallel lines cut by a transversal.		3.25	0.64	2.06
F2	Understand and apply the conditions sufficient to establish the congruence of two triangles.		2.92	0.27	1.69
F3	Understand and use the Pythagorean Theorem.		3.92	0.91	2.50
F4	Prove simple geometric results.		2.65	0.14	1.56
		Category Mean	**3.18**	**0.49**	**1.95**

G	**Understand and use the Geometry of Circles.**		**2.65**	**0.32**	**1.19**
G1	Apply the basic properties of a chord of a circle.		2.13	0.23	0.94
G2	Apply the basic properties for angles of figures inscribed in a circle.	Mainly inscribed triangles.	2.08	0.18	0.81
G3	Apply some basic properties of tangents to a circle.		2.56	0.18	0.88
		Category Mean	**2.26**	**0.20**	**0.88**

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
H	**Understand and use Triangle Trigonometry.**		**3.83**	**0.45**	**0.69**
H1	Understand and apply the primary trig ratios as related to triangles.		3.83	0.32	0.75
H2	Know the trig ratios for special angles as numerical values.	0°,30°,45°, 60°,90°	3.46	0.27	0.56
H3	Use formulae involving trig ratios for the area of a triangle.		2.40	0.18	0.38
H4	Use the Laws of Sines and Cosines.		3.10	0.14	0.13
H5	Understand and apply the concept of a vector.		1.98	0.45	0.13
H6	Determine the resultant of two vectors.		1.94	0.32	0.06
		Category Mean	**2.78**	**0.28**	**0.33**
J	**Understand and use Data Analysis.**		**0.79**	**2.34**	**1.38**
J1	Organise and graph data from an experiment.		0.88	2.43	1.50
J2	Obtain, organise and graph sample data.		0.83	2.34	1.19
J3	Apply sample information to population information.		0.54	2.20	0.81
J4	Determine 90% confidence intervals.		0.42	2.02	0.38
J5	Select appropriate sampling methods.		0.38	2.07	0.56
		Category Mean	**0.61**	**2.21**	**0.89**
K	**Understand and use Sequences and Series.**		**2.75**	**1.55**	**1.00**
K1	Understand and use arithmetic sequences and series formulae.		2.75	0.98	1.00
K2	Understand and use geometric sequences and series formulae.		2.71	0.98	0.81
K3	Understand and use sigma notation.		2.92	1.95	0.50
K4	Apply the formula for an infinite geometric series.	$S = a/(1–r)$	2.63	0.93	0.50
		Category Mean	**2.75**	**1.21**	**0.70**
L	**Understand and use the Function Concept.**		**4.00**	**1.77**	**1.88**
L1	Understand and use function notation.		4.00	1.93	1.88
L2	Identify relations that are not functions.		3.73	0.98	1.50
L3	Graph basic $\mathbf{R \rightarrow R}$ functions like linear, absolute value, etc.	On Cartesian plane.	3.83	1.61	1.50
L4	Understand the functional transformations: translation, reflection, expansion w.r.t. the function's expression.	w.r.t = with respect to	3.29	0.86	0.63
L5	Determine the inverse of a function.	If it exists.	3.67	0.70	0.06
		Category Mean	**3.70**	**1.22**	**1.11**

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
M	**Understand and use Straight Line and Linear Functions.**		**3.96**	**3.64**	**2.44**
M1	Graph and write the equations for linear functions.		4.00	3.59	2.31
M2	Understand and use the slope of a line.		4.00	3.64	2.31
M3	Write direct variations as equations.		3.48	2.16	1.44
		Category Mean	**3.83**	**3.13**	**2.02**
N	**Understand and use the Quadratic Function.**		**3.83**	**1.50**	**1.13**
N1	Graph and write the equations for given quadratic functions.		**3.75**	**1.20**	**0.94**
N2	Determine the max. or min. value of a quadratic function.	Without calculus.	2.98	0.93	1.00
		Category Mean	**3.36**	**1.07**	**0.97**
P	**Understand and use Quadratic Relations.**		**3.29**	**0.55**	**0.50**
P1	Work with the graph and the equation (+ features) of a circle.		3.31	0.50	0.44
P2	Work with the graph and the equation (+ features) of parabolas.		3.17	0.50	0.38
P3	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred at the origin.		2.85	0.41	0.19
P4	Work with the graph and the equation (+ features) of ellipses and hyperbolas centred away from the origin.		2.58	0.32	0.19
P5	Resolve simple analytic geometry situations—midpoint, distance between points, point(s) of intersection.		3.38	0.59	0.13
		Category Mean	**3.06**	**0.46**	**0.26**
Q	**Understand and use Polynomial Functions.**		**3.42**	**0.68**	**0.44**
Q1	Graph and analyse polynomial functions of various degrees.	2 and greater.	3.31	0.61	0.25
Q2	Apply the Factor and Remainder Theorems.		2.48	0.18	0.06
Q3	Understand and use notation for complex numbers.		1.79	0.14	0.13
Q4	Analyse the nature of the roots for some polynomial functions.		2.52	0.18	0.00
		Category Mean	**2.53**	**0.28**	**0.11**

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
R	**Understand and use the Logarithmic Function.**		**3.79**	**1.30**	**0.63**
R1	Write exponential expressions in logarithmic form.		3.79	1.20	0.63
R2	Determine values of exponential and logarithmic functions.		3.79	1.16	0.56
R3	Graph and analyse logarithmic functions of base *a*.	*a* is an integer >1.	3.42	0.52	0.38
R4	Understand and apply the properties of logarithms.		3.67	0.93	0.44
R5	Use common or natural logarithms to evaluate expressions.	Base 10 and base *e*.	3.63	1.11	0.44
R6	Perform calculations found in practical settings using the techniques of logarithms.	Appl'ns: forgetting, sound, etc.	3.23	0.89	0.56
		Category Mean	**3.59**	**0.97**	**0.50**

S	**Understand and use Circular Trigonometric Functions.**		**3.92**	**0.18**	**0.19**
S1	Understand and use the primary trig ratios as related to angles of rotation.		3.79	0.18	0.19
S2	Identify reference angles.		3.54	0.18	0.13
S3	Understand and use the radian measure for angles.		3.79	0.14	0.25
S4	Understand and use the 3 reciprocal trig functions.	Cot(θ), Sec(θ), Csc(θ)	3.17	0.09	0.13
S5	Graph and analyse the functions sine, cosine and tangent.		3.79	0.18	0.06
S6	Determine the period and amplitude from the equation or graph of a sine, cosine or tangent function.		3.21	0.09	0.00
S7	Using its graph or equation, determine the phase-shift of a sine, cosine or tangent function.		2.83	0.09	0.00
S8	Use the basic trig identities to re-write trig expressions.		3.46	0.09	0.07
S9	Understand and use sum, difference and double angle identities.		3.38	0.09	0.00
S10	Prove simple identities.		3.00	0.09	0.00
S11	Use trigonometric identities to simplify trig expressions.		3.38	0.09	0.00
		Category Mean	**3.39**	**0.12**	**0.07**

T	**Understand and use some Concepts of the Calculus.**		**1.06**	**0.86**	**0.06**
T1	Use limit notation and determine the limit value of given functions at a point.		1.10	0.77	0.00
T2	Determine whether a given sequence has a limit.		1.10	0.50	0.00
T3	Determine the derivative of a given polynomial function.		0.94	0.52	0.00
T4	Understand and use the power, product and quotient rules.		0.94	0.57	0.00
		Category Mean	**1.02**	**0.59**	**0.00**

Category Code	Proficiency Descriptor	Comment	Importance Rating		
			Calc	Intro Stats	MFEE
U	**General, Higher Level or Specialised Proficiencies.**		**n/a**	**n/a**	**n/a**
U1	Find and write up solutions to word problems connected to specific math/stats topics being studied.		3.54	3.07	2.63
U2	Engage in the completion of extended projects involving multiple, varied areas of mathematics/statistics.		1.85	2.25	1.31
U3	Be familiar with writing simple proofs.		2.50	1.02	1.00
U4	Understand and apply the concept of set, subset and complement of a set.		2.00	2.89	2.31
U5	Use the set operations: union, intersection.		2.06	2.93	2.44
U6	Graph set interrelations using a Venn Diagram.		1.52	2.84	2.38
U7	Use a scientific calculator.	Log, Trig, etc.	3.58	2.73	1.31
U8	Use a graphing calculator.		2.21	0.93	0.31
U9	Use a computer algebra system.	Maple, Mathematica etc.	1.42	0.52	0.13
U10	Use a statistical software package.	SPSS, Minitab, etc.	0.17	1.82	0.19

The Information Sheet (below), for respondents, was intended to outline the intent of and to answer some questions about the Survey.

Mathematics Proficiencies Survey 1 – INFORMATION SHEET

Where did this initiative originate?

One of the Programs of the **Centre for Curriculum, Transfer & Technology** (C2T2) concerns **Secondary to Post-Secondary Transitions**. The Secondary to Post-Secondary Transitions team was asked to investigate the possibility of detailing post-secondary course/program prerequisites in a manner that, among other benefits, would provide a greater incentive for secondary students to choose appropriate courses and to be more diligent in preparing for their chosen post-secondary programs.
To perform this investigation, C2T2 identified four disciplines/programs in which to conduct pilot projects—Mathematics, English, Health & Human Services and Business Administration.

Who oversees this Project?

John Meagher, Senior Program Manager, Secondary to Post-Secondary Transitions, is the Project Manager. In that rôle, John is also Chair of the Mathematics Proficiencies Steering Committee which is comprised of representatives from BCAMT, Ministry, ABE, BCcupm, and post-secondary institutions.

Where is it all expected to lead?

These projects are very much exploratory in nature. Depending on the recommendations resulting from

the projects and the will (and ability) of C2T2 to take the outcomes to the next steps, we could expect to increase the amount of information given to secondary teachers/counselors for use with their students and we might also witness an effect on the confidence with which prerequisites to post-secondary are stated.

What is a *proficiency*?

The word ***proficiency*** was chosen to describe a discipline-related skill, competence or ability that, in normal circumstances, a student should possess in order for one reasonably to expect successful involvement in the *next* level or area of study. Although proficiencies can be stated in very general or quite specific terms, we have, as a first effort of this kind, attempted to achieve a balance between vague generality and tedious detail. It is noteworthy that the proficiency descriptors that have been used in this Survey refer to learning typically acquired at the 10^{th}, 11^{th} or 12^{th} grade level.

What about general background proficiencies?

Clearly, students entering post-secondary programs are expected to have acquired general knowledge about the world around them, to be able to read/write, and to have certain understandings and skills relative to mathematics (number, shape, patterns, etc.). Although such 'general background' is vital to a student's successful performance in a post-secondary mathematics/statistics course, we make no attempt to identify such general background proficiencies in this Survey.

In the Survey, how are Proficiency Descriptors intended to be interpreted?

A Proficiency Descriptor like "Factor binomials and trinomials", on its own, out of context, would require a great deal of additional information about the kind and size of coefficients, the range of degrees allowed, the sort of factors that would be accepted and so on. An exhaustive definition of each proficiency would, at this stage, needlessly encumber the survey and would make further documentation completely unwieldy. It is expected, therefore, that respondents to the Survey will read Proficiency Descriptors as relating to course objectives as found in a grade 11 or 12 course outline. If the Proficiency Category has relevance to your course and the Proficiency Descriptor has meaning for you, then we ask that you rate its importance to you. A *next step* may involve the creation of specific examples for those proficiencies whose importance mathematics instructors rate highly.

Why are there three Surveys?

The Mathematics Proficiencies Steering Committee felt that the proficiencies needed from secondary school for the various areas of mathematics and statistics in the post-secondary system are unique to those areas. The proficiencies needed for a student to succeed in linear algebra differ from those required to do well in statistics. In order to make the Survey manageable, yet as meaningful as possible, the Steering Committee directed that it should separately investigate **Calculus, Introductory Statistics** and **Mathematics for Elementary Education**.

Which <u>Calculus</u> stream is the Survey about?

The Mathematics Proficiencies Steering Committee suggested that no "stream" designation be put on the **Calculus** survey. The feeling was that, for purposes of this first survey, respondents should seek identify and rate general, pre-calculus proficiencies. If the mathematical proficiency is required for first-year

calculus in any of your "streams", then it should be identified and rated.

What does Introductory Statistics mean?

The BCcupm's subcommittee on **Introductory Statistics** has struggled with this question. **Introductory Statistics** appears to describe a statistics course that is suitable for any student having taken no previous, post-secondary statistics course. Such courses typically demand little, if any, mathematical sophistication beyond that acquired in secondary school. A first course in statistics that requires, and builds on, two semesters of calculus would, on the other hand, not be classed as an **Introductory Statistics** course for purposes of this Survey.

What if your institution does not offer a Mathematics for Elementary Education course?

If your College, University or Institute does not offer a **Mathematics for Elementary Education** course, then please return the blank Survey form indicating this fact. You would do the same if you did not offer a **Calculus** or an **Introductory Statistics** course.

Where can you obtain more information?

For more information, please e-mail Leo Neufeld at: neufeld@camosun.bc.ca

APPENDIX D

Contacts - Mathematics Proficiencies Survey

Calculus Contacts

Tasoula Berggren Simon Fraser University	Peggy Tilley/Norman Eyres Camosun College
John Byl Trinity Western University	Ted Bentley Capilano College
Charles W. Lamb University of British Columbia	Nicolas Buck and colleagues College of New Caledonia
Sam Walters University of Northern British Columbia	Kim Chew Columbia College
Denton Hewgill University of Victoria	Erich Durnberger Coquitlam College
David Sabo British Columbia Institute of Technology	Jim Bailey College of the Rockies
Veda Abu-Bakare Open Learning Agency	Susan Oesterle Douglas College
Avtar Sadhra Kwantlen University College	David Lidstone Langara College
Patrick Ng Malaspina University-College	Slava Simice North Island College
Edward Butz Okanagan University College	Mohamed Chabi Northern Lights College
Kirk Evenrude University College of the Cariboo	Mona Izumi Northwest Community College
Susan Milner and colleagues University College of the Fraser Valley	Mac Sinclair Selkirk College

Introductory Statistics Contacts

Robin Insley Simon Fraser University	Bill Calver/Susan Chen Camosun College
Don Ariel Trinity Western University	Ted Bentley Capilano College
James V. Zidek University of British Columbia	Ted Biech and colleagues College of New Caledonia
Vladimir Vinogradov University of Northern British Columbia	Grace Chiu Coquitlam College
Roger R. Davidson University of Victoria	Leslie Molnar College of the Rockies
Louise Routledge British Columbia Institute of Technology	Dan Henschell Douglas College
Veda Abu-Bakare Open Learning Agency	Cheryl McKeeman Langara College
Jim Gunson Kwantlen University College	Richard Stride North Island College
Ray Sproule Malaspina University-College	Mohamed Chabi Northern Lights College
Edward Butz Okanagan University College	Mona Izumi Northwest Community College
Shane Rollans University College of the Cariboo	Mac Sinclair Selkirk College
David Chu/J.B. Garner University College of the Fraser Valley	

Mathematics for Elementary Education Contacts

Malgorzata Dubiel Simon Fraser University	Jill Britton Camosun College
Richard J. Sutcliffe Trinity Western University	Judy Malcolm College of New Caledonia
William E. Pfaffenberger University of Victoria	Leslie Molnar College of the Rockies
Veda Abu-Bakare Open Learning Agency	Susan Oesterle Douglas College
Allison Nishihara Kwantlen University College	Slava Simice North Island College
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Don DesBrisay University College of the Cariboo	Mona Izumi Northwest Community College
Jane Cannon University College of the Fraser Valley	John Josafatow Selkirk College

APPENDIX E

Mathematics Proficiencies Steering Committee

John Meagher, Senior Program Manager, Secondary to Post-Secondary Transitions, Centre for Curriculum Transfer and Technology (Steering Committee Chair)	Clint Lee Department of Mathematics & Statistics Okanagan University College
Ian Bailey Department of Mathematics & Statistics Malaspina University-College	David Leeming Department of Mathematics & Statistics University of Victoria
Robert Adams Department of Mathematics University of British Columbia	Jean MacLeod Mathematics Department Vancouver Community College
Jack Bradshaw Department of Mathematics & Statistics University College of the Cariboo	Susan Milner Department of Mathematics & Statistics University College of the Fraser Valley
Ron Coleborn, President British Columbia Association of Mathematics Teachers	Leo Neufeld, Chair British Columbia Committee on the Undergraduate Programme in Mathematics
Malgorzata Dubiel Department of Mathematics & Statistics Simon Fraser University	Susan Oesterle Department of Mathematics & Statistics Douglas College
John Inglis Department of Mathematics Kwantlen University College	Malcolm Sneddon Centre for Applied Academics Field Services Branch

APPENDIX F

Resources

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