Report to the BCcupm:

First-Year Core Calculus

May 2002

Core Calculus Subcommittee:

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Acknowledgments

We sincerely thank Kwantlen University College and Simon Fraser University for their generous provision of ideal meeting spaces for the Core Calculus Subcommittee. Conveniently located in supportive environments, these accommodations assisted significantly in promoting productivity and success at our sessions.

Without the consent of the mathematics/statistics departments represented by Core Calculus Subcommittee members, this initiative could not have begun. We are grateful to each department for its understanding and support in providing a strong, willing participant for the work of the committee.

Not least, we acknowledge the enthusiastic support and assistance given to this undertaking by the BC Council on Admissions & Transfer. BCCAT and, particularly, the staff members in the BCCAT Office have cheerfully and competently offered expert advice and assistance throughout the process. We cordially thank them for their vision and guidance.

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Executive Summary

This initiative, whose goal is to ease the transferability of first-year calculus courses within BC, was formally launched by the BC Committee on the Undergraduate Programme in Mathematics (BCcupm) at its meeting in May 2001. An *ad hoc* committee of the BCcupm was formed to "develop a list of core topics" for first-year calculus streams and was asked to report to its May 2002 meeting.

Activities

The BC Council on Admissions & Transfer (BCCAT) approved a Transfer Innovations Project (TIP) application to support financially the Subcommittee's activities. This TIP grant enabled the Subcommittee to meet on five occasions gain a clear, mutual understanding of the committee's task, to develop efficiently the core calculus curricula proposal and to agree on the recommendations for the BCcupm.

Findings

- ✓ It was found that, in the first-year calculus courses examined, the number of topics common to all was proportionately quite large.
- ✓ When topic placement in first-year calculus courses was compared by semester, considerably less agreement in topic order was observed.
- ✓ In deciding on a set of topics for inclusion in a first-year calculus core, the Subcommittee learned that an "enabling transfer" approach, as opposed to a prescriptive approach, was essential to any expectation of general agreement.
- ✓ Although a core of topics for all first-year calculus courses exists, a marked difference in emphasis was noted between the Sciences Calculus and the Social Sciences/Business Calculus streams.

The Core Calculus Curricula

The Subcommittee developed a Core Curriculum for first-year Sciences Calculus (page9) and first-year Social Sciences/Business Calculus (page10). To accommodate differing emphases locally, without compromising existing standards, these curricula have been presented in two parts, Core Topics and Additional Topics, whose relative weightings have been specified.

Recommendations

The Subcommittee drafted a proposal for the BCcupm (page11) containing first-year core curricula for Sciences Calculus and for Social Sciences/Business Calculus courses, and implementation statements to be considered for adoption.

Background and Objectives

Background

The British Columbia Committee on the Undergraduate Programme in Mathematics (BCcupm) is the post-secondary mathematics and statistics articulation committee for the province. Its activities are conducted under the auspices of the BC Council on Admissions and Transfer (BCCAT). Articulation committees were established to foster and maintain course/program transfer arrangements to ease student movement from one educational institution to another.

At various times in the past, members of the BCcupm had advanced the thought that shaping calculus courses, particularly at some of the smaller institutions, could be significantly assisted by the existence throughout the province of greater uniformity in course content for these courses. If this were the case, not only would such institutions be helped, but students transferring from one post-secondary institution to another would also become direct beneficiaries. Thus, at its May 2001 meeting in Cranbrook, the BCcupm passed the following motion:

> **That, in order to ease transferability across all first-year calculus streams, we strike a subcommittee to develop a list of core topics for all streams of Calculus I and Calculus II.**

To implement this motion, the Meeting struck an *ad hoc* committee with representation from universities, university-colleges and colleges, and directed this subcommittee to prepare a report for the May 2002 meeting of the BCcupm. Further, to realistically support this initiative, the Meeting recommended that financial assistance be sought to ensure a reasonable round of committee activities.

Objectives

The overall goal of the Subcommittee was to accomplish the directive of the BCcupm, i.e., to develop a core curriculum for all streams of first-year calculus. To achieve this, the Subcommittee would need to attain the following objectives:

- Come to a mutual agreement concerning the primary mandate of the committee.
- Identify general issues and challenges that related to the committee's task.
- Compare the existing first-year calculus curricula for the institutions represented by members of the committee.
- Identify a set of core topics in the streams of first-year calculus.
- Develop core curricula for first-year calculus courses that would serve, in the first instance, as standards for course transfer.
- Consult during the process, where possible and as widely as prudent, with colleagues regarding the findings and the proposals of the committee.
- Prepare a report of the committee's findings and proposals for consideration by the BCcupm.

Activities

Following the impetus given this initiative, the Chair of the BCcupm proceeded immediately with the following tasks:

a) Subcommittee membership - confirmed the nominees.
b) Communication - established a means of contact among all members (electronic mail was chosen).
c) Initial Meeting - found a suitable site and a mutually convenient time.
d) Financial Support - explored the possibilities of obtaining support from BCCAT for the Core Calculus initiative and completed the application process.

Meetings of the Subcommittee were held with agendas and action plans as indicated:

Meeting (July 17, 2001, at Kwantlen University College (Richmond))

Agenda: Orientation, Clarifying Objectives/Tasks, Identifying Issues, Agreeing on Strategies, Delegating Tasks and Appointing a Chair.

Major Outcomes: Copies of current calculus course curricula for member institutions were available at the Meeting – Preliminary discussion of the Sciences Calculus with particular reference to one institution (UBC) – Bruce Kadonoff was appointed Chair – Members were to explore curricular options suggested and consult with colleagues (locally and in their constituency) prior to the next meeting.

Meeting (October 30, 2001, at Simon Fraser University (Harbour Centre))

Agenda: TIP Requirements, Reports of feed-back from colleagues, Identifying a Sciences Calculus Core (spreadsheet), Issues of the Social Sciences/Business Stream and Review of the objectives of this initiative.

Major Outcomes: Approval of a TIP grant and its implications for this initiative were announced – Reports on preliminary reactions from colleagues as to directions proposed – Significant progress on the Sciences Core – Clarification of the project's objective: to ease calculus course transfer for students.

Meeting (January 29, 2002, at Simon Fraser University (Harbour Centre))

Agenda: Consideration of Feedback from Colleagues, Draft of Recommendations to BCcupm, Work on the Social Sciences/Business Calculus Core, Advice on the Interim Report to BCCAT.

Major Outcomes: Decisions on suggestions from colleagues regarding topics for inclusion in the Calculus Core – Preliminary decisions on proposed weighting ratios for Core/Additional Topics – Suggestion that a set of sample questions be constructed – Consideration of draft recommendations for BCcupm – Discussion of an outline of the Interim Report to BCCAT on the TIP.

Meeting (March 19, 2002, at Simon Fraser University (Harbour Centre))

Agenda: Review of present status of the Core Calculus Curricula, Review of the Material Samples provided by Bruce Kadonoff, Refinement of the Recommendations for BCcupm, Discussion of the Final Report to BCCAT.

Major Outcomes: Decisions on place for the Material Samples in the Subcommittee's reports – Amplification of the recommendations statement for the BCcupm – Eliciting feedback from colleagues on the draft recommendations statement – Clarifying the character of the reports to BCcupm and to BCCAT.

Meeting (May 7, 2002, at Simon Fraser University (Harbour Centre))

Agenda: Receiving feedback on consultation with Colleagues, Final reviews of the Recommendations to BCcupm and the Core Calculus Curricula, Information and decisions regarding the completion of the Report to BCcupm.

Outcomes: Final decisions on the Recommendations to BCcupm and the Core Calculus Curricula – Directions concerning the Report to BCcupm – Discussion of the reporting session at the May Meeting of BCcupm.

In addition to their other professional duties and outside of committee meeting sessions, members of the Subcommittee devoted considerable time to such tasks as preparing draft documents, analyzing course outlines, reviewing support materials and consulting with colleagues. Each member attended every meeting and members always came to these sessions prepared to report feedback from their constituency and to move forward.

Findings

After having made informal comparisons of all the first-year calculus courses offered by each of the institutions represented by committee members, the committee focused on identifying the common topics for these courses at Simon Fraser University, The University of BC and the University of Victoria. The topics common to courses at these institutions are recorded in Appendices A and B. Further, the two streams chosen for this comparison were the Sciences Calculus and Social Sciences/Business Calculus. The Sciences Calculus courses contained more common topics than did those of Social Sciences/Business Calculus.

Besides examining the principal issue, that of the difficulties faced by mathematics departments attempting to prepare students for transfer to a number of other post-secondary educational institutions, the Subcommittee also explicitly identified some general considerations worth noting as work proceeded. Although narrowly not in its mandate, the committee realized that a mutual understanding of these related issues and challenges was critical to any expectations for success.

Some of the issues considered were:

- There is an inter-flow of students between the sciences and the non-sciences programs even within individual post-secondary institutions.
- Post-secondary institutions are autonomous.
- Prerequisites for entry to present calculus courses are not consistent throughout the province.
- In calculus course delivery, there are differences in the sequence in which topics are delivered from institution to institution.
- Practices with respect to the inclusion of technology both in instruction and in evaluation differ within institutions and among them.
- Can core curricula be so described as to avoid a gradual degeneration of some existing calculus programs? Maintaining standards is vital.

In preparing to formulate a set of core topics for first-year calculus, the committee came to the following agreements:

a) The core calculus curricula developed by the committee would be described for first-year courses. Recommendations on how to divide these into semester courses would not be made.

b) Given the almost universal practice in the BC post-secondary system of having separate calculus courses for the Sciences and for the Social Sciences/Business, the committee determined also to treat these individually.

c) The committee felt that the primary purpose for establishing core curricula in calculus was to ease the course transfer difficulties that beset institutions in developing appropriate courses and that frustrate students whose calculus backgrounds fail to meet expectations in follow-on courses.

The committee's main activity was the development of the Core Curricula that appear next in this report. However, throughout the process and between committee meetings, members took time to consult and confer with colleagues both at their own institutions and with others in their constituency. The feedback received was valuable and, although respondents would like to have expressed opinions on a final product, indicated the existence of general interest in and support for this initiative.

The diversity of perspectives among members of the committee was an important factor in achieving a balanced set of core curricula with a clear proposal.

When stepping away from the detail associated with the activities of the Subcommittee, it is noteworthy that they bear an eerie similarity to those occurring regularly in most mathematics departments when developing courses for large student populations.

Core Curriculum – Sciences Calculus
First-Year

A first year (two-semester) Sciences Calculus course must include all the topics from the **Core Topics** list. It is expected that coverage of this material would constitute three-quarters of the course(s) with the remaining one-quarter chosen from the **Additional Topics** list. For breadth, at least four Additional Topics should be included.

Reference Text: Edwards & Penney, *Calculus, Early Transcendentals, Fifth Edition*, Prentice Hall,1998.

Core Topics (75%)

1. Limits, continuity, intermediate value theorem
2. Differentiation
   - First and second derivatives with geometric and physical interpretations
   - Mean value theorem
   - Derivatives of exp and log functions, exponential growth and decay
   - Derivatives of trigonometric functions and their inverses
   - Differentiation rules (including chain rule, implicit differentiation)
   - Linear approximations and Newton's Method
   - Optimization - local and absolute extrema with applications
3. Taylor polynomials and special Taylor series (sin, cos, exp, 1/(1-x))
4. Curve sketching
5. Integration
   - Definition of the definite integral
   - Areas of plane regions
   - Average value of a function
   - Fundamental Theorem of Calculus
   - Integration techniques: substitution (including trig substitution), parts, tables, partial fractions
   - Applications of integration
6. Numerical Integration (including The Trapezoidal Rule)
7. Improper integrals: evaluation and convergence estimates
8. Differential equations (first-order linear) with applications

Additional Topics (25%)

1. Sequences and series
2. Arc length, volumes, centroids, surface areas
3. Additional differential equations topics
4. Complex numbers
5. Continuous probability density functions
6. Polar coordinates and parametric equations (with calculus applications)
7. Additional numerical methods (eg. Simpson's Rule)
8. Related rates
9. L'Hôpital's Rule

Core Curriculum – Social Sciences/Business Calculus
First-Year

A first year (two-semester) Social Sciences/Business Calculus course must include all the topics from the **Core Topics** list. It is expected that coverage of this material would constitute approximately two-thirds of the course(s) with the remaining one-third chosen from the **Additional Topics** list. For breadth, at least four Additional Topics should be included.

Reference Text: Haeussler and Paul, *Introductory Mathematical Analysis for Business, Economics, and the Life and Social Sciences*, *Ninth Edition*, Prentice Hall, 1998.

Core Topics (67%)

1. Limits, continuity, intermediate value theorem
2. Differentiation
   - First and second derivatives with geometrical and physical interpretations
   - Applications to economics, business and social sciences
   - Derivatives of exp and log functions, exponential growth and decay with applications
   - Derivatives of trigonometric functions
   - Differentiation rules (including chain rule, implicit differentiation)
   - Linear approximations and Newton's Method
   - Optimization - local and absolute extrema with applications
3. Curve sketching
4. Integration
   - Definition of the definite integral
   - Areas
   - Average value of a function
   - Fundamental Theorem of Calculus
   - Integration techniques: substitution, parts, tables
   - Applications of integration
5. Numerical integration (including The Trapezoidal Rule)
6. Differential equations (first-order linear) with applications

Additional Topics (33%)

1. Introduction to probability and statistics
2. Partial derivatives and Lagrange multipliers
3. Matrix analysis and Gaussian Elimination
4. Sequences and series
5. Arc length, volumes, centroids, surface areas
6. Taylor polynomials and special Taylor series (sin, cos, exp, 1/(1-x))
7. Improper integrals: evaluation and convergence estimates
8. Continuous probability density functions
9. Related rates
10. Derivatives of inverse trigonometric functions
11. Further techniques of integration
12. Additional numerical integration methods

Transfer Proposal for First-Year Calculus

The BCcupm affirms the autonomy of BC's post-secondary institutions in their freedom to design calculus courses to meet the needs of their unique constituencies. However, the diversity of calculus courses in first year offerings in these institutions has created difficulties for transferring students and their institutions. For example, while current first-year "business" calculus courses at SFU, UBC and UVic share many common topics, the additional material covered by these institutions is irreconcilable within a single two-term calculus sequence.

This proposal addresses the significant challenges encountered by students transferring first-year calculus courses and by their sending institutions. To lessen the impact of these challenges, we propose that all post-secondary institutions in BC recognize a common curriculum for first-year calculus courses. Such recognition will have the following benefits for students and their institutions:

a) Provide transferring students with a solid background for subsequent math courses requiring first-year calculus.
b) Allow primarily sending institutions to design calculus courses that will meet the needs of their students post-transfer.
c) Guide primarily receiving institutions in assessing the adequacy of courses proposed for transfer.

Recommendations:

1. That the BCcupm accept the Report of the First-Year Calculus Sub-committee and endorse the Core Sciences Calculus and Core Social Sciences/Business Calculus curricula as described in this report.

2. That receiving institutions grant full transfer credit to first-year calculus courses from other BC post-secondary institutions whose courses are consistent with the curricula as described in this report.

3. That, when designing or modifying first-year calculus courses, all BC post-secondary mathematics departments strive to include within their courses the calculus topics as described in this report.

4. That any post-secondary institution sensing that the Core Calculus curricula as described in this report require a full or partial review raise its concerns at the next regularly scheduled meeting of the BCcupm.

5. That, in the absence of an earlier full review, the Core Calculus curricula be subject to a mandatory, full review after five years.

Appendix A - Sciences Calculus - Included Topics Comparison

Topic	Edwards & Penney	SFU Math151	SFU Math152	UBC Math100	UBC Math101	UVic Math100	UVic Math101	All
Functions and graphs	Chap 1			Yes				
Limits and Continuity	Chap 2	Yes		Yes		Yes		✔
Differentiation	Chap 3	Yes		Yes		Yes		✔
Additional Applications	Chap 4	Yes		Yes		Yes		✔
Integration								
Antiderivatives	5.2	Yes				Yes		
The integral test	5.3		Yes		Yes	Yes		✔
Riemann Sums	5.4		Yes		Yes	Yes		✔
Evaluation of Integrals	5.5		Yes		Yes	Yes		✔
Fundamental Theorem	5.6		Yes		Yes	Yes		✔
Substitution	5.7		Yes		Yes	Yes		✔
Areas of planes	5.8		Yes		Yes	Yes		✔
Numerical integration	5.9		Yes		Yes	Yes		✔
Applications								
Setting up integrals	6.1		Yes		Yes		Yes	✔
Volumes	6.2,6.3		Yes		Yes		Yes	✔
Arc length	6.4		Yes		Yes		Yes	✔
Separable ODE	6.5		Yes		Yes		Yes	✔
Force and Work	6.6		Yes		Yes			
Exp and Log functions	Chap 7	Yes		Yes		Yes		✔
Linear 1st order ODE	7.6				Yes	Yes		
Transcendental functions								
Inverse trig functions	8.2	Yes		Yes			Yes	✔
L'Hôpital's rule	8.3,8.4	Yes		Yes			Yes	✔
Hyperbolic functions	8.5	Yes					Yes	
Techniques of Integration	Chap 9		Yes		Yes		Yes	✔
Polar Coordinates								
Conic sections	10.1,10.2	Yes					Yes	
Area computations in polar	10.3		Yes				Yes	
Parametric curves	10.4	Yes					Yes	
Integration in parametrics	10.5		Yes				Yes	
Infinite Series								
Infinite Sequences	11.2		Yes				Yes	
Convergence	11.3		Yes				Yes	
Taylor Series	11.4		Yes	Yes			Yes	✔
The integral test	11.5		Yes					
Comparison tests	11.6		Yes					
Alternating series	11.7		Yes				Yes	
Power series	11.8 11.9		Yes	Yes			Yes	✔
Probability and Statistics	Handouts				Yes			
Ordinary Diff Equations								
Separable ODE	Handouts				Yes			
2nd order homogeneous	Handouts				Yes			
Non-homogeneous	Handouts				Yes			
Variation of parameters	Handouts				Yes			
Applications	Handouts				Yes			
Complex numbers	Handouts		Yes		Yes			

Appendix B - Social Sciences/Business Calculus - Included Topics Comparison

Topic	Haeussler & Paul	SFU Math157	SFU Math158	UBC Math104	UBC Math105	UVic Math102	UVic Math103	All
Non-Calculus Topics								
Functions review	Chap 3 and 4	Yes		Yes		Yes		✔
Exp and log review	Chap 5	Yes		Yes		Yes		✔
Matrix algebra	Chap 6		Yes				Yes	
Linear Programming	Chap 7		Yes					
Compound interest and PV	8.1 8.2			Yes		Yes	Yes (+8.3)	
Probability and Statistics	9.1 - 9.4, 10.1,10.2				Yes			
Limits and Continuity	Chap 11	Yes		Yes		Yes		✔
Differentiation								
Definition	12.1,12.2	Yes		Yes		Yes		✔
Rates of change	12.3	Yes		Yes		Yes		✔
Continuity	12.4	Yes		Yes		Yes		✔
Product and Quotient	12.5	Yes		Yes		Yes		✔
Chain and Power	12.6	Yes		Yes		Yes		✔
Der of log and exp	13.1,13.2	Yes		Yes		Yes		✔
Implicit diff	13.3	Yes		Yes		Yes		✔
Logarithmic diff	13.4	Yes						
Higher order	13.5	Yes		Yes		Yes		✔
Der of trig functions	Ed & Pen 3.7			Yes		Yes		
Der of Inv trig functions	Ed & Pen 8.2			Yes				
Taylor Series	Ed & Pen 11.4			Yes				
Curve Sketching	Chap 14	Yes		Yes		Yes		✔
Applications								
Applied max/min	15.1	Yes		Yes		Yes		✔
Differentials	15.2	Yes		Yes		Yes		✔
Elasticity of demand	15.3	Yes		Yes				
Newton's Method	15.4			Yes				
Integration								
Indefinate integration	16.1		Yes		Yes	Yes		✔
Initial conditions	16.2		Yes		Yes	Yes		✔
Subsitution	16.3,16.4		Yes		Yes	Yes		✔
Definite integral	16.5,16.6		Yes		Yes	Yes		✔
Fundamental theorem	16.7		Yes		Yes	Yes		✔
Area between curves	16.8,16.9		Yes		Yes	Yes		✔
Applications of area	16.10		Yes		Yes	Yes		✔
Integration by parts	17.1		Yes		Yes			
Integration by partial fraction	17.2		Yes		Yes			
Integration by tables	17.3		Yes		Yes			
Average value	17.4		Yes		Yes	Yes		✔
Approximation (linear)	17.5		Yes		Yes			
Simpson's Rule	17.5		Yes		Yes			
Improper Integrals	17.8		Yes		Yes			
Continuous random variables	18.1,18.2				Yes			
Differential Equations	17.6,17.7		Yes		Yes	Yes		✔
Multivariable Calculus								
Partial derivatives	19.1,19.2	Yes			Yes		Yes	✔
Applications	19.3	Yes			Yes		Yes	✔
Implicit	19.4	Yes			Yes		Yes	✔
Higher order	19.5	Yes			Yes		Yes	✔
Chain Rule	19.6	Yes			Yes		Yes	✔
Max / min in two variables	19.7	Yes			Yes		Yes	✔
Lagrange multipliers	19.8	Yes			Yes		Yes	✔
Lines of regression	19.9	Yes						

Appendix C - The Core Calculus Subcommittee

The Core Calculus Subcommittee was struck by the BCcupm at its May 2001 meeting in Cranbrook, British Columbia, as an *ad hoc* committee to "develop a set of core topics" for first-year calculus courses. The committee's membership was to include representation as follows: one each from Simon Fraser University, The University of BC, the University of Victoria, a university college and a college. The Chair of the BCcupm was to act as a facilitator and as an *ex officio* member of the committee.

The First-Year Core Calculus Subcommittee members were:

Name	Institution	Transfer Perspective
Dr. Rustum Choksi	Simon Fraser University	Receiving
Bruce Kadonoff (Chair)	Coquitlam College	Sending
Dr. David Leeming	University of Victoria	Receiving
Dr. Philip Loewen	The University of BC	Receiving
Dr. Casey McConill	Kwantlen University College	Sending
Leo Neufeld (TIP Contractor)	BCcupm	Sending