BCCUPMS Calculus Readiness Test

MAY 17,2017

Goal

- To develop a 30-item multiple choice diagnostic test to predict success/failure in first-semester calculus courses

Test items based on the BCCUPMS Mathematics Proficiencies Project Report (Leo Neufeld)

Category Code	Sub-Proficiency Descriptor	Comment	Importance Rating
L1	Understand and use function notation.		**4.00**
M1	Graph and write the equations for linear functions.		**4.00**
M2	Understand and use the slope of a line.		**4.00**
A1	Simplify polynomial expressions.		**3.96**
A2	Add and subtract polynomial expressions.		**3.96**
A3	Multiply and simplify polynomial expressions.		**3.96**
C1	Use the Laws of Exponents to simplify expressions.		**3.96**
E1	Solve linear equations.		**3.96**
C2	Write radical expressions in exponential form.		**3.92**
F3	Understand and use the Pythagorean Theorem.		**3.92**
C3	Apply the Laws of Exponents to expressions with rational exponents.		**3.88**
E2	Solve linear inequalities		**3.85**

Questions given in a free-response format (LON-CAPA) to students at

College of the Rockies (J. Bailey)
Douglas College (W. Snider)
Langara College (N. Franzova)
UNBC (E. Beveridge)

Incorrect answers with high frequency used as distractors for the multiple choice versions of these questions

1		solve_trigonometric_equations_A.problem					
2		0					
3		Response 11					
4	userna	Submission	Correct	Award Detail	Time	Attempt	Awarded
126	lang10	{5pi/12,7pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Mar 27 01:0	1	
127	doug3(	{5pi/4,7pi/4}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sat Feb 14 09:1	1	
128	doug3(	{5pi/4,7pi/4}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Mar 5 10:1	1	
129	doug3(	{5pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Mar 6 01:0	1	
130	lang10	{7pi/12, 11pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Mon Feb 16 11	1	
131	doug3(	{7pi/12, pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Feb 22 10:	1	
132	doug3(	{7pi/12,11pi/12,17pi/12,23pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Mar 6 04:1	1	
133	cotrVa	{7pi/12,11pi/12,17pi/12,23pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Sep 25 12:	1	
134	doug3(	{7pi/12,11pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Wed Feb 25 01	1	
135	lang10	{7pi/12,11pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Mar 15 01:	1	
136	lang10	{7pi/6, -pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Feb 19 11:	1	
137	lang10	{7pi/6, 11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Tue Feb 10 10:	1	
138	lang10	{7pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Tue Feb 10 12:	1	
139	doug3(	{7pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Feb 15 12:	1	
140	doug3(	{7pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Mon Feb 23 06	1	
141	lang10	{7pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Mar 22 12:	1	
142	lang10	{7pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Mon Feb 16 05	1	
143	doug3(	{pi,2pi}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Feb 26 10:	1	
144	cotrRei	{pi/12, 5pi/12}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Sep 11 08:	1	
145	doug3(	{pi/12, 5pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Mar 5 11:	1	
146	cotrLou	{pi/2, 2*pi}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Sep 28 11:	1	
147	cotrZur	{pi/2,pi,3pi/2,2pi}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Sep 26 02:2	1	
148	cotrKn	{pi/2}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Sep 26 08:2	1	
149	lang10	{pi/3, 4pi/3}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Apr 3 04:52	1	
150	doug3(	{pi/3,2pi/3,pi/2,3pi/2,0,pi,pi/6,5pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Feb 20 04:2	1	
151	cotrDe	{pi/3,5pi/3,-7pi/3,-11pi/3}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Sep 28 08:	1	
152	doug3(	{pi/4, 3pi/4, 5pi/4, 7pi/4}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Thu Feb 26 10:	1	
153	cotrCra	{pi/4,3pi/4,5pi/4,7pi/4}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Sep 26 09:5	1	
154	lang10	{pi/6,11pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Apr 5 05:3	1	
155	doug3(	{pi/6,5pi/6}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Fri Mar 6 01:3	1	
156	lang10	{x+pi/4}	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Wed Feb 11 01	1	
157	cotrBe	0 2*pi	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Wed Sep 17 09	1	
158	doug3(	11*ï€/ 12	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Sun Feb 22 12:	1	
159	doug3(	2pi	{7pi/12, 11pi/12, 19pi/12, 23pi/12}	INCORRECT	Mon Feb 23 10	1	

Find all solutions of $2\sin 2x = -1$ over the interval $[0, 2\pi)$.

Solution Set:

- $\left\{ \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4} \right\}$
- $\left\{ \frac{7\pi}{6}, \frac{11\pi}{6} \right\}$
- $\left\{ \frac{7\pi}{12}, \frac{11\pi}{12}, \frac{19\pi}{12}, \frac{23\pi}{12} \right\}$
- $\left\{ \frac{7\pi}{12}, \frac{11\pi}{12} \right\}$
- $\left\{ \frac{4\pi}{6}, \frac{5\pi}{6} \right\}$

Analysis of the distractors was used to introduce randomization into the questions

Ana Culibrk
Jim Bailey
Natasha Davidson
Michael Nyenhuis
Wayne Broughton

Threaded View Chronological View Other Views ...
Export Undelete all deleted entries

Jim Bailey (BAILEY@cotr.bc.ca:sfu) Hide Delete Reply Submissions (Sun Jun 12 09:13:18 am 2016 (PDT))

$\{\frac{7\pi}{12}, \frac{11\pi}{12}\}$: the solution on $[0, \pi)$.

$\{\frac{4\pi}{6}, \frac{5\pi}{6}\}$: $\frac{5\pi}{6}$ is a solution of $2\sin x = 1$; I don't know about $\frac{4\pi}{6}$.

$\{\frac{7\pi}{6}, \frac{11\pi}{6}\}$: the solutions to $2\sin x = -1$.

$\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$: rather bizarre.

trig equations Natasha Davidson (davidsonn@douglascollege.ca:sfu) Hide Delete Reply Submissions (Mon Jun 13 11:14:51 am 2016 (PDT))

the answer pi/4, 3pi/4, 5pi/4, 7pi/4 is to the equation |sin(2x)| = 1, so drop the factor of 2 and solve for -/+ 1...

4pi/6 is a solution to 2cos(2x) = -1 but the combination doesn't give me an algorithm

Re: trig equations Michael Nyenhuis (Michael.Nyenhuis@kpu.ca:sfu) Hide Delete Reply Submissions (Mon Jun 13 06:12:51 pm 2016 (PDT))

One possibility for 4pi/6 is dropping a perpendicular to the y axis rather than the x, which effectively changes the sine to a cosine.

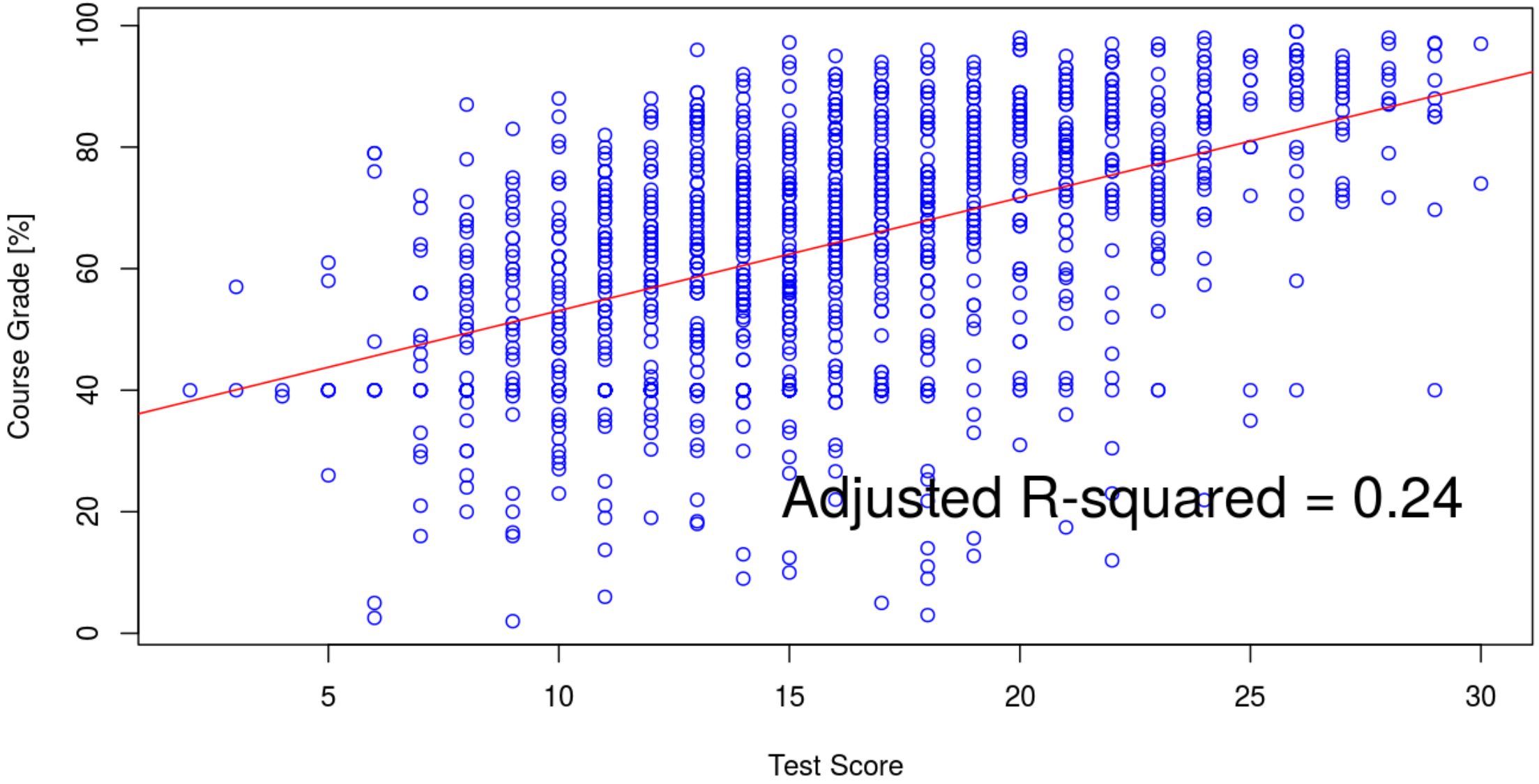
During the 2016 Fall semester, the Calculus Readiness Test was administered in the following courses (J. Bailey, M. Nyenhuis, J. Butterfield)

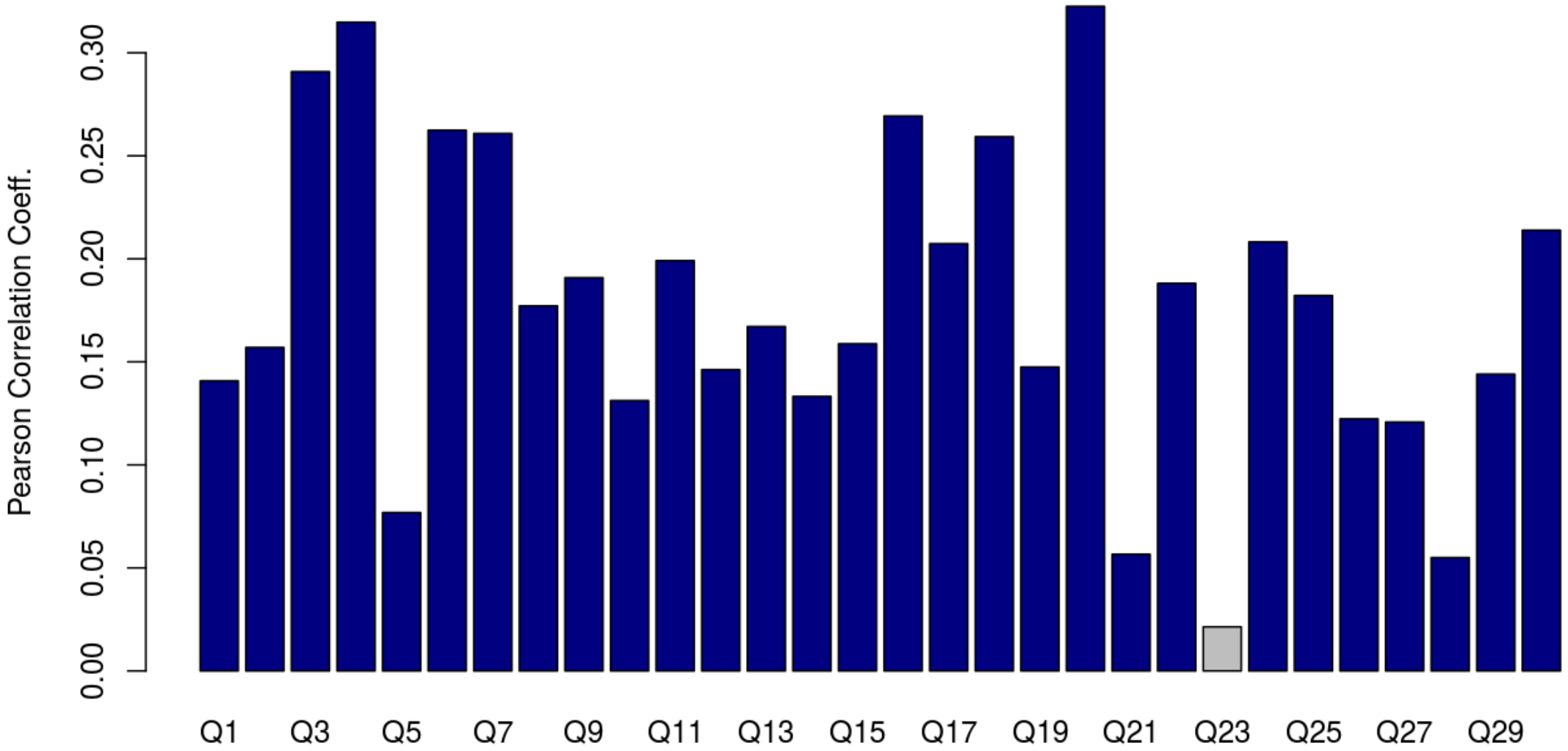
CotR MATH 103 (28 students)
KPU MATH 1120 (30 students)
KPU MATH 1130 (31 students)
UVic MATH 100 (691 students)
UVic MATH 109 (436 students)

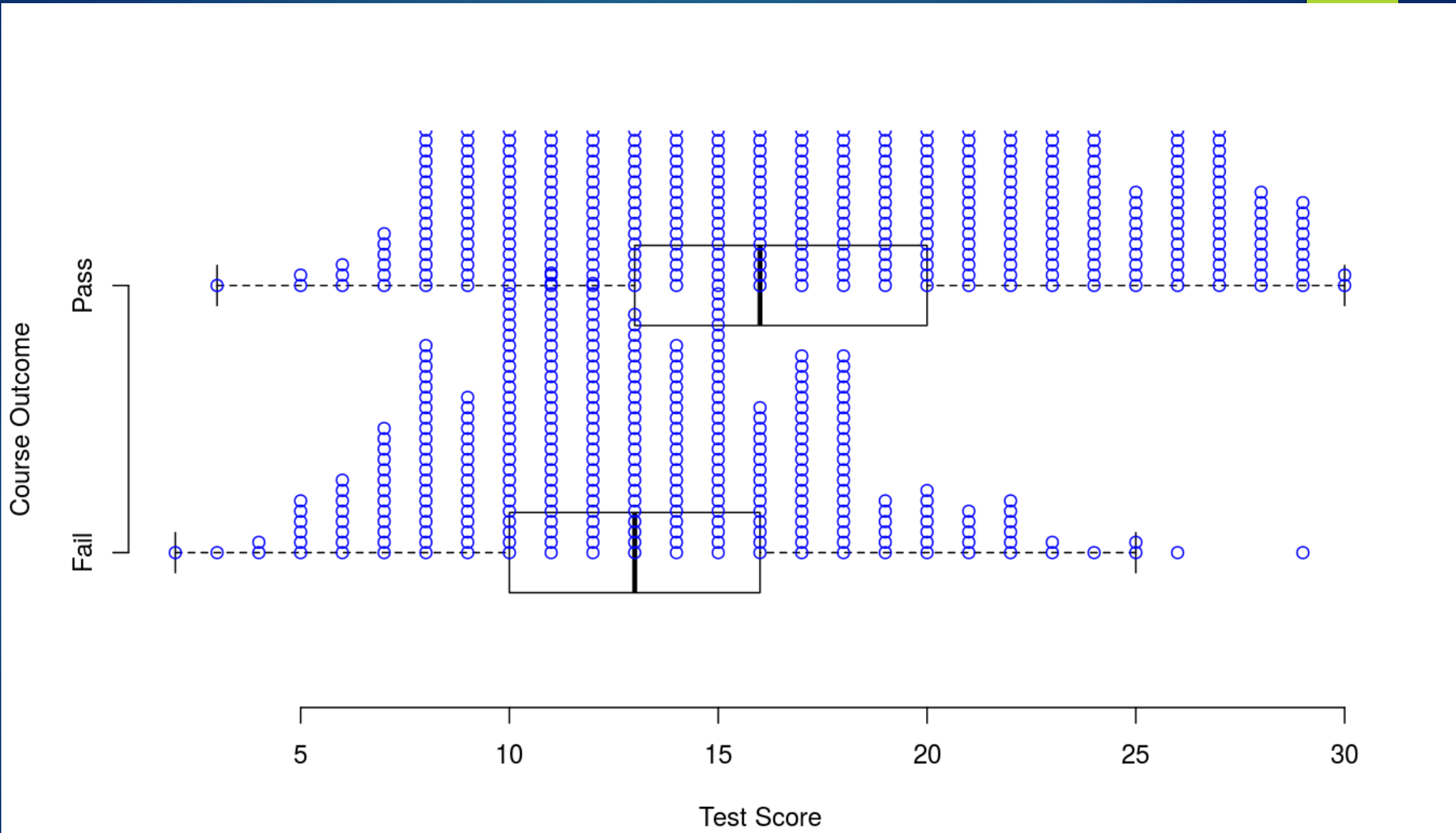
Final grades were compared with overall CRT scores and also responses to individual test items using

TRU Math & Stats: Readiness Tests Analysis (Richard Taylor)
http://legendre.tru.ca/readiness_tests/

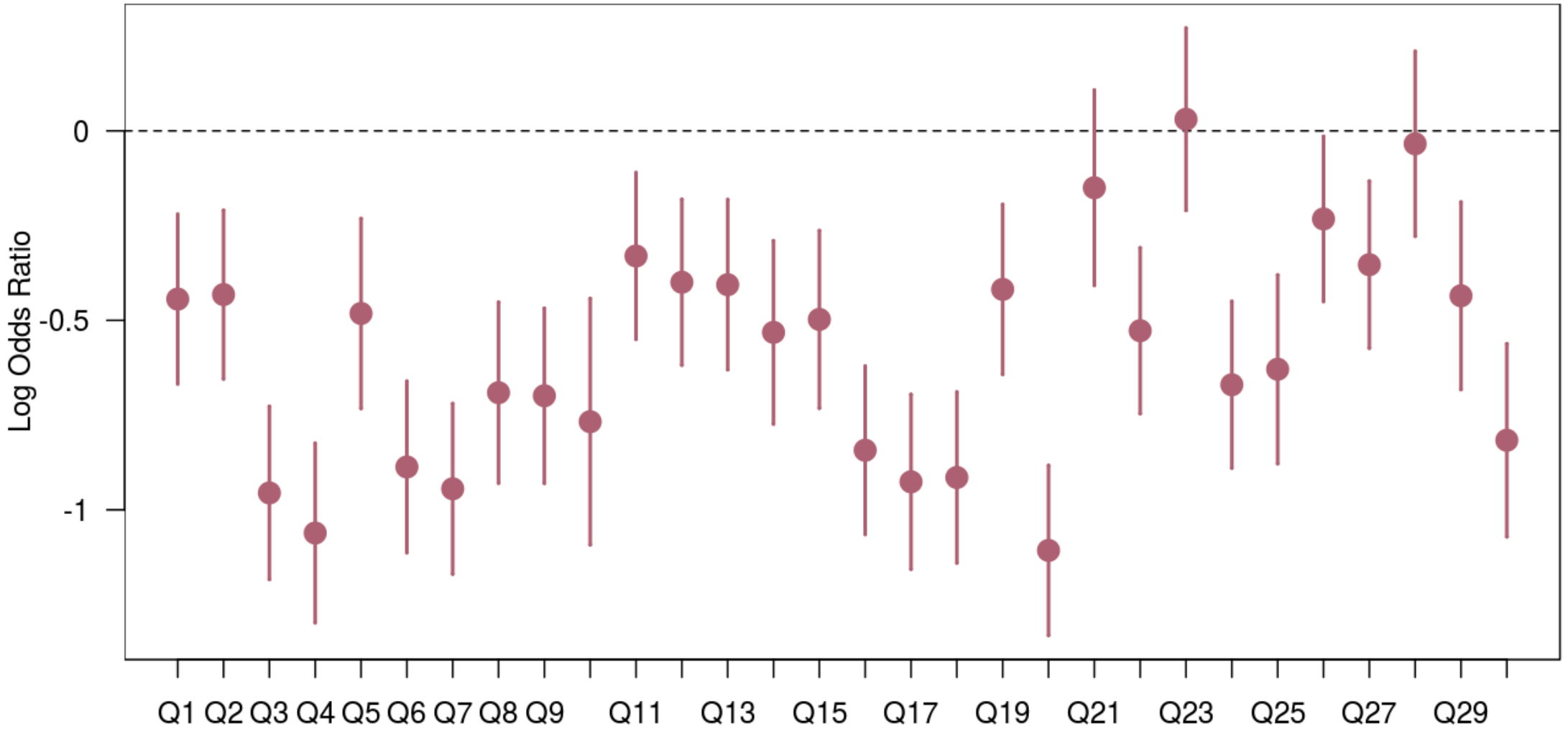
The following charts use the combined data for all five courses (all had a pass rate of 50%).


Course Grade [%]
0
20
40
60
80
100
5
10
15
20
25
30
Test Score
Adjusted R-squared = 0.24

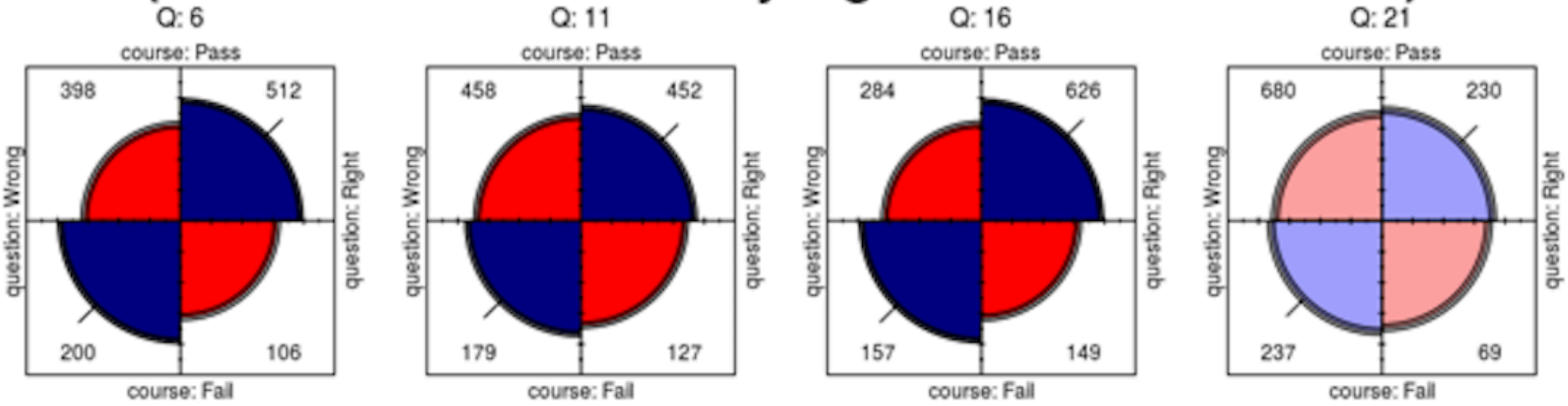

Correlation of Question Response With Course Grade
(blue => statistically significant positive correlation)
Pearson Correlation Coeff.
0.00
0.05
0.10
0.15
0.20
0.25
0.30
Q1
Q3
Q5
Q7
Q9
Q11
Q13
Q15
Q17
Q19
Q21
Q23
Q25
Q27
Q29

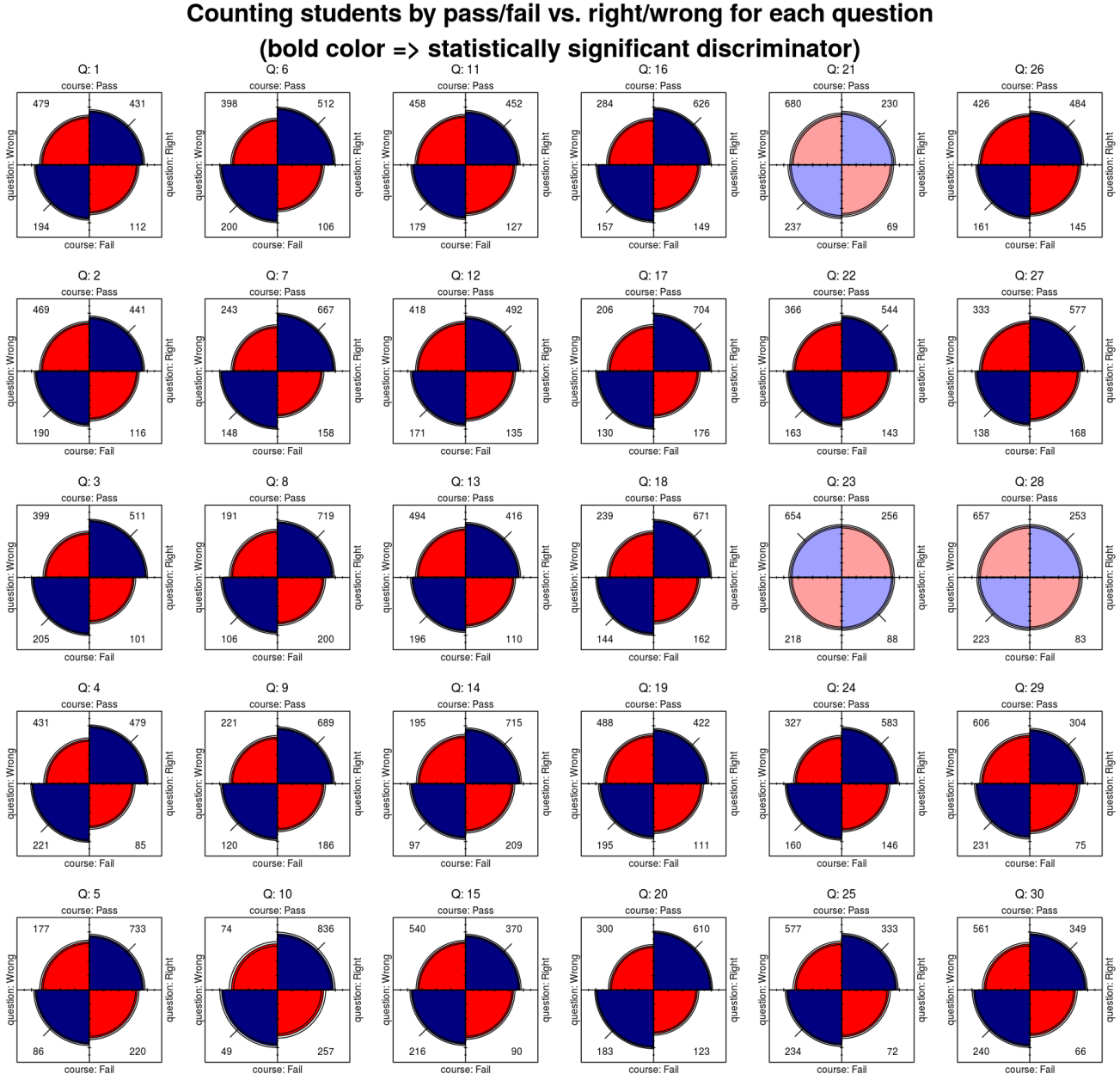

Course Outcome
Pass
Fail
5
10
15
20
25
30
Test Score

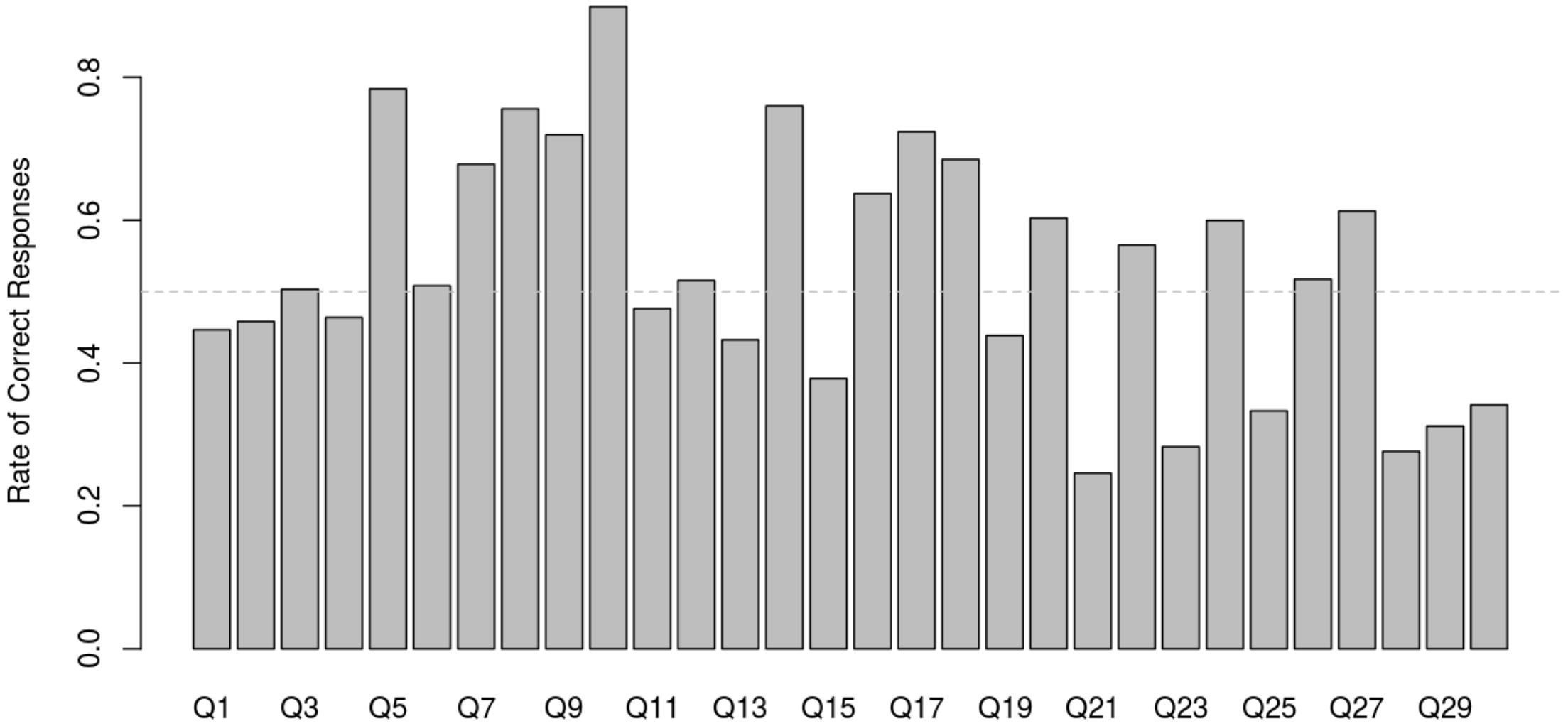
(log of the) odds ratio, i.e. the ratio of the odds of failing the course if answering correctly to the odds of failing if answering incorrectly.

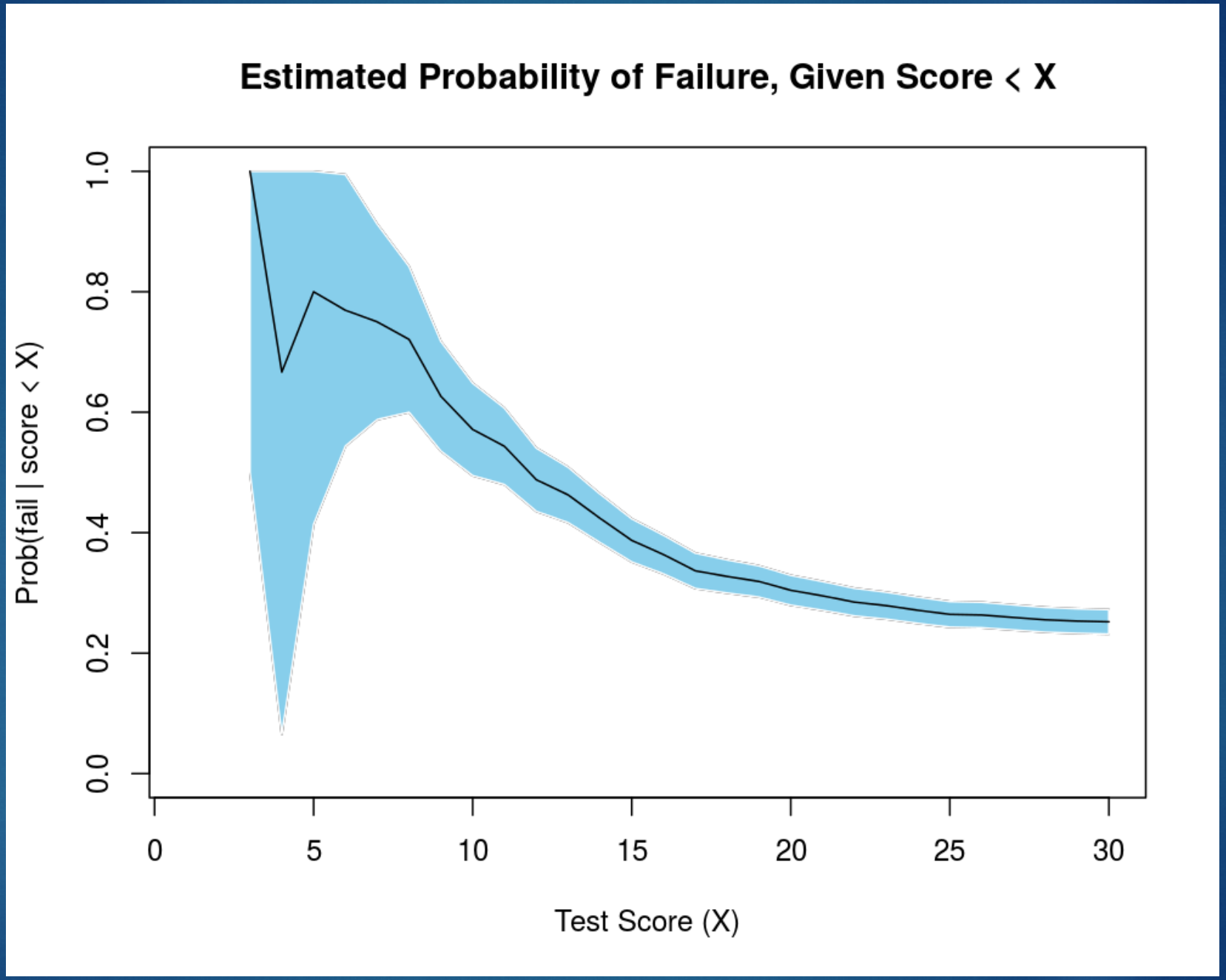

Log Odds Ratio
0
-0.5
-1
Q1 Q2 Q3 Q4 Q5 Q6 Q7 Q8 Q9 Q11 Q13 Q15 Q17 Q19 Q21 Q23 Q25 Q27 Q29

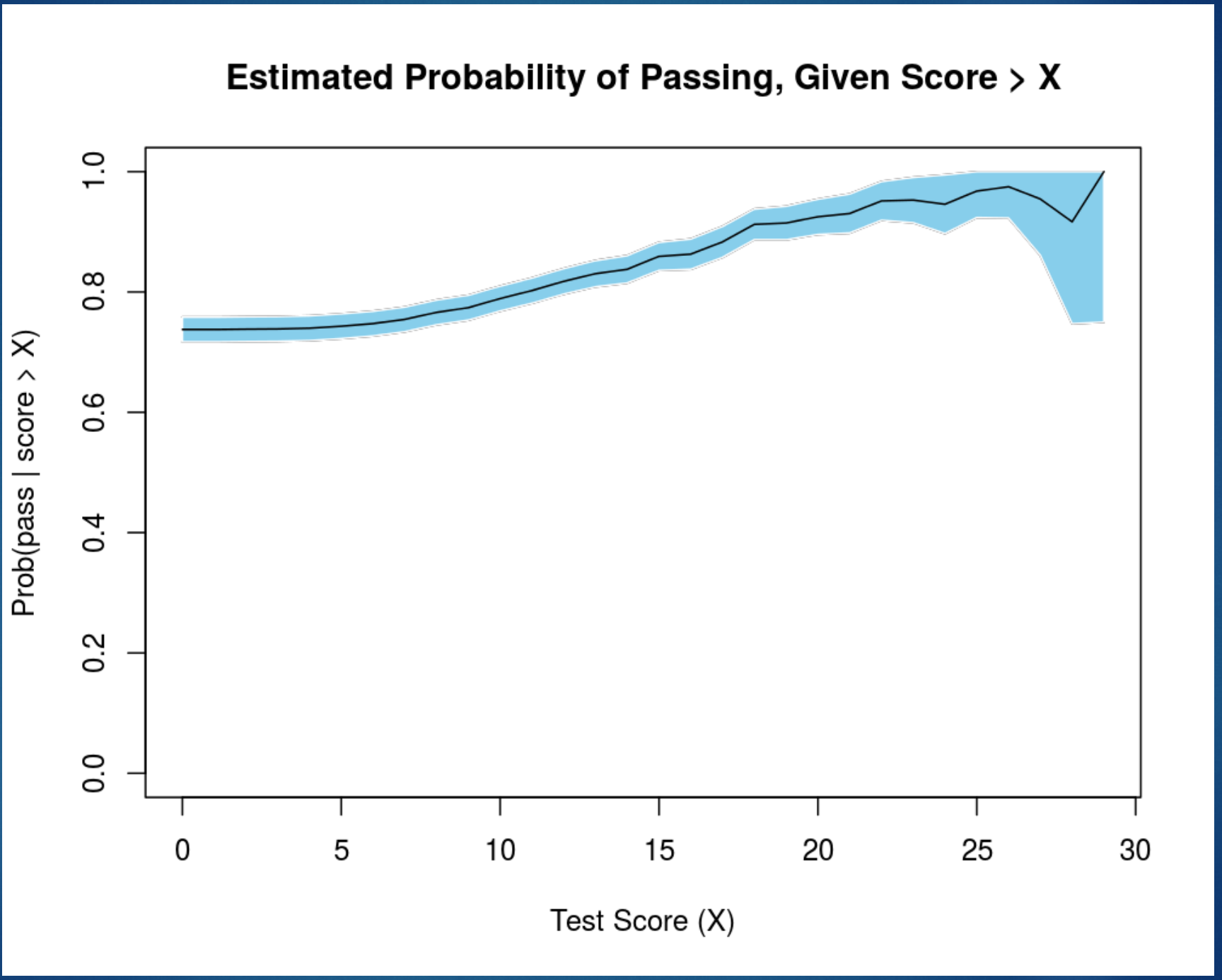
Counting students by pass/fail vs. right/wrong for each question (bold color => statistically significant discriminator)




Counting students by pass/fail vs. right/wrong for each question
(bold color => statistically significant discriminator)
Q: 1
course: Pass
479
431
question: Wrong
question: Right
194
112
course: Fail
Q: 6
course: Pass
398
512
question: Wrong
question: Right
200
106
course: Fail
Q: 11
course: Pass
458
452
question: Wrong
question: Right
179
127
course: Fail
Q: 16
course: Pass
284
626
question: Wrong
question: Right
157
149
course: Fail
Q: 21
course: Pass
680
230
question: Wrong
question: Right
237
69
course: Fail
Q: 26
course: Pass
426
484
question: Wrong
question: Right
161
145
course: Fail
Q: 2
course: Pass
469
441
question: Wrong
question: Right
190
116
course: Fail
Q: 7
course: Pass
243
667
question: Wrong
question: Right
148
158
course: Fail
Q: 12
course: Pass
418
492
question: Wrong
question: Right
171
135
course: Fail
Q: 17
course: Pass
206
704
question: Wrong
question: Right
130
176
course: Fail
Q: 22
course: Pass
366
544
question: Wrong
question: Right
163
143
course: Fail
Q: 27
course: Pass
333
577
question: Wrong
question: Right
138
168
course: Fail
Q: 3
course: Pass
399
511
question: Wrong
question: Right
205
101
course: Fail
Q: 8
course: Pass
191
719
question: Wrong
question: Right
106
200
course: Fail
Q: 13
course: Pass
494
416
question: Wrong
question: Right
196
110
course: Fail
Q: 18
course: Pass
239
671
question: Wrong
question: Right
144
162
course: Fail
Q: 23
course: Pass
654
256
question: Wrong
question: Right
218
88
course: Fail
Q: 28
course: Pass
657
253
question: Wrong
question: Right
223
83
course: Fail
Q: 4
course: Pass
431
479
question: Wrong
question: Right
221
85
course: Fail
Q: 9
course: Pass
221
689
question: Wrong
question: Right
120
186
course: Fail
Q: 14
course: Pass
195
715
question: Wrong
question: Right
97
209
course: Fail
Q: 19
course: Pass
488
422
question: Wrong
question: Right
195
111
course: Fail
Q: 24
course: Pass
327
583
question: Wrong
question: Right
160
146
course: Fail
Q: 29
course: Pass
606
304
question: Wrong
question: Right
231
75
course: Fail
Q: 5
course: Pass
177
733
question: Wrong
question: Right
86
220
course: Fail
Q: 10
course: Pass
74
836
question: Wrong
question: Right
49
257
course: Fail
Q: 15
course: Pass
540
370
question: Wrong
question: Right
216
90
course: Fail
Q: 20
course: Pass
300
610
question: Wrong
question: Right
183
123
course: Fail
Q: 25
course: Pass
577
333
question: Wrong
question: Right
234
72
course: Fail
Q: 30
course: Pass
561
349
question: Wrong
question: Right
240
66
course: Fail


Rate of Correct Responses
0.0
0.2
0.4
0.6
0.8
Q1
Q3
Q5
Q7
Q9
Q11
Q13
Q15
Q17
Q19
Q21
Q23
Q25
Q27
Q29


Estimated Probability of Failure, Given Score < X
Prob(fail | score < X)
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30
Test Score (X)


Estimated Probability of Passing, Given Score > X
Prob(pass | score > X)
0.0
0.2
0.4
0.6
0.8
1.0
0
5
10
15
20
25
30
Test Score (X)

Remarks

Carrying out this analysis on the larger classes individually gives different results:

UVic MATH 100: Adj. R-sq = 0.29
UVic MATH 109: Adj. R-sq = 0.18

Some problems should be replaced, in particular,
Q21, Q23, Q28
Most students do poorly on these questions regardless of whether they pass or fail calculus.

It is difficult to propose a clear recommendation from the data at this stage.