Perspective based on experience & biases

(Disclaimer: No surprises…..except our lack of focus and action)

Pre-calculus at colleges and universities: based on curriculum developed, by mathematics specialists, and evolved over decades, aided by textbooks that pretty much agree on topics, and sequencing, for the preparation of calculus.

Math 12 (soon to be called pre-calculus): Has also evolved but is more of a hodge-podge of topics, some not necessary for the preparation of calculus. Less depth, gaps in sequencing, and largely prepared and vetted by ministry reps and consultants with varying mathematical backgrounds and experiences, with texts thrown together to match the 'curriculum du jour'.

Large Issues, basically what's missing:

1. No concept of proof ….. given, statement of required to prove, proof process. In throwing out Euclidean proofs we've thrown out 2 babies with the bathwater
2. Geometry: What's been put back in is incomplete with much of it going into the non-precalc streams. Congruency of triangles, needed for example, to prove the slope relationship for perpendicular lines. Similar triangles, for trigonometry. The distance formulas (1D & 2D), necessary for equation of a circle, which also helps explain replacing x by (x-h) for translations.
3. One-to-one functions, necessary for inverse functions (especially restricted domains)
4. Polynomial and rational inequalities: e.g. find the domain of $y = \log\frac{1+x}{1-x}$
5. Sigma notation: perhaps explaining why $\sum_{k=1}^{n}\frac{3}{10^k} \to \frac{1}{3}$ as $n \to \infty$
6. Casual limits, without the notation, around behavior of functions around asymptotes
7. Expansion of binomials, using Pascal's triangle: might help with $(a+b)^2 \neq a^2 + b^2$
8. Conics a.k.a quadratic relations
9. Methodology issues, e.g. multiple methods and representations to strengthen concepts e.g. solving trig equations with graphs and reference triangles
10. Constructing functions from geometric situations
11. Using function notation for slope, i.e., the difference quotient: What is $f(x+h)$?
12. Much more needed on natural logarithms.
13. Systems of equations involving quadratics, including sketching graphs
14. Language (not missing but serious): equation vs. expression, solve vs. simplify
15. Piecewise functions, including piecewise definition of $|x|$

Algebraic mistakes usually traceable to misusing the fundamental laws of operations

1. $a\left(\frac{b}{c}\right)=\frac{ab}{ac}$ similarly confusion with $\frac{\frac{a}{b}}{c}$
2. $\frac{a+b}{c}=\frac{a}{c}+b$ This is a violation of the distributive law which should be taught in the texts as $\frac{a+b}{c}=\frac{1}{c}(a+b)=c^{-1}(a+b)=\frac{a}{c}+\frac{b}{c}$
3. $-x=-1(x)$ seems innocuous but real troubles with $-\frac{a+b}{c}=\frac{-1(a+b)}{c}$
4. $\frac{a}{b}=0\rightarrow a=0, b\neq 0$, this rule should be taught with the $ab=0$ rule

Confusing input/output for functions – especially trig functions

1. $f(x)=x^2+x-3$, Evaluate $f(3)$ compared to solve $f(x)=3$. This becomes more poignant if the given function is a piecewise function. **Domain vs Range**
2. Solve $\cos x=0$ compared to evaluate $\cos 0$

Use of formulas – especially when substitution is advisable

1. Use $\sin 2\theta=2\sin\theta\cos\theta$ to find an expression for $\sin 3x\cos 3x$
2. Need to think of expressions as numbers when substituting: $\sin^2 x+\cos^2 x=a^2+b^2$
3. Given a line in the first quadrant, with positive intercepts, and a point $P(x,y)$ on the line, find the area of shaded rectangles/triangles as a function of ***x***

Multiple representations – and their interdependence

1. Relating the algebraic solution of a polynomial equation to the x-intercepts of the corresponding function, and sketching the graph of that function. Conversely, given x-ints of a cubic function $(-2,0),(-1,0),(3,0)$ and y-int. $(0,12)$, find the function. Students get stuck in a process driven mode and miss these connections.
2. Connecting the 3 representations of the equation $\cos x=\frac{1}{2}$. The (unit) circle definition of the cosine, the graph of the function, and a table of values for the function. Then relate all this to what the function-based calculator supplies.

Fractions! Fractions! Fractions!

What's ½ of 4/5? … or if we need another….What's the number half-way between $\frac{\pi}{4}$ *and* $\frac{\pi}{2}$